Development of Mathematical Model to Study the Variation Temperature due to the Collision of Air Molecules or Aerosol Particle with Aircraft

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ABSTRACT

To study the variation of temperature with the collision of the air molecules, with aircraft the authors consider the random velocities of the air molecules or aerosol and aircraft. After this, the authors start to develop and extend the old relation of gases, collision, velocities, and temperature. On combining, the authors derived a new equation and relation among these parameters. The derivation for this work assumes some suitable and considerable, assumptions based on collision theory. The relation shows, on collision between the aircraft and air molecules the velocity of the aircraft does not change while the velocity of the air molecules goes on change, which is our goal.

The change in velocity of gases is used to develop and study the variation of temperature of the atmosphere, the variation of temperature takes place because the speed of aircraft and air molecules are exchangeable in some cases. Therefore, two relations are developed (24) and (25), finally, which depend upon the initial velocities of aircraft and air molecules. On considering the case for initial velocities of aircraft is greater than initial velocities of air molecules, after the collision, the velocities of gases go on the increase, and hence the temperature increase and vice-versa in case of initial velocities of aircraft is less than the initial velocity of air molecules.

Keywords: Air Molecules, Collision, Velocity, Temperature, Aircraft, Aerosol.

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I. INTRODUCTION

The expansion of solid and liquid are not affected significantly by pressure alone, while the expansion of gas is greatly affected. There are various laws of gas based on its temperature, pressure, and volume. The two special gases for our study are Boyle's law and Charle's law. Boyle's law state at constant temperature $PV = \text{constant}$, and Charle's law at constant pressure called the law of volume and represent as $V = V_0(1 + \gamma_p\theta)$, where V_0 the volume of gas at 0°C and V at $\theta^\circ\text{C}$, γ_p is a constantly called coefficient of expansion of the gas at constant pressure. Also, at constant volume called the law of pressure and represent as $p = p_0(1 + \gamma_v\theta)$, where p_0 the volume of gas at 0°C and p at $\theta^\circ\text{C}$, γ_v is a constantly called coefficient of expansion of the gas at constant volume.

These two laws can be combined to obtain a general relationship between pressure, volume, and temperature called ideal gas equation as $PV = nRT$, whose behavior is independent of attractive and repulsive forces.

A. Expression for Kinetic Energy of Masses

Consider a body of mass m , initially at rest (i.e., initial velocity is zero) and final velocity v then the gains kinetic energy equal to the work done by the force. Therefore, authors can write $E = F \times s$, where s is distance travel. From Newton law of motion, $F = ma$ and kinetic energy $K.E = \frac{1}{2}mv^2$, authors can also extend this relation as:

$$mas = \frac{1}{2}mv^2$$

$$v = \sqrt{2as} \quad (1)$$

B. Relation of Kinetic Energy and Temperature during the Collision

Temperature is a measurement of the average kinetic energy of the molecules in an object or a system. Kinetic energy is the energy that an object has because of its motion. The molecules in a substance have a range of kinetic energies because they do not all move at the same speed. As a substance absorbs heat the particles move faster so the average kinetic energy and therefore the temperature

increase. A gas is made up of a large number of extremely small particles (molecules or atoms) in constant, random, straight-line motion. Molecules occupy very little volume (most of the container is free space) and collide with one another, without any interaction force between the molecules. The presence of molecules can gain or lose energy on collision, but the total energy remains constant.

Let us consider a mixture of gases having N identical molecules of mass m , in considering volume (V), with each molecule has a speed (u) but different for different molecules, then the pressure due to the number of collisions with consideration of average speed of different molecules is given by the relation:

$$P = \frac{N}{3V} \sum_i^N m_i \bar{u}_i^2 \quad (2)$$

Here $\bar{u}_i$ = the average of the square of speeds of gases molecules, on arranging this equation (2),

$$PV = \frac{1}{3} N m \bar{u}_i^2 \quad (3)$$

For the development of temperature relation with kinetic energy for an average speed of molecules can be written as $\frac{1}{2} m \bar{u}_i^2$. The kinetic energy of a mole of molecules can be written as $E = \frac{1}{2} N_A m \bar{u}^2$ (where N_A is Avogadro's number), therefore one can relate E , N_A , m and $\bar{u}_i$ as:

$$m \bar{u}^2 = \frac{2E}{N_A} \quad (4)$$

Now from (3) and (4), the obtained relation is expressed as:

$$PV = \frac{2}{3} nE \quad (5)$$

Here, $n = \frac{N}{N_A}$, since $PV = nRT$ ideal gas equation, therefore equation (5) given a relation becomes:

$$K.E = E = \frac{3}{2} RT \quad (6)$$

This shows that the temperature of the gas is the measure of the kinetic energy of the molecules.

On considering the velocity of air there are 3 types of velocity mean velocity (u_m), average velocity (u_{av}), and root mean square velocity (v_{rms}). These are all are related to $\sqrt{T}$, directly and their ratio is to each other as 0.82:0.92:1, respectively.

II. MATERIAL AND METHOD

The atmosphere is a mixture of gases with a major composition of 78% nitrogen, 21% oxygen, and 1% other gases, such as argon or helium. The concentration of gases is to even minute pressures because of the lack of strong molecular cohesion. Also, gases like carbon dioxide, nitrous oxides, methane, and ozone are trace gases that account for about a tenth of one percent of the atmosphere. The unique

concentration of water in the atmosphere varies from 0-4%, which very important in predicting the weather.

CO₂ has a historical background was measured with non-dispersive infrared (NDIR), which is one of the most important greenhouse gases, and can absorb infrared radiation and play an important role in global warming. Another method of measuring the concentration of the CO₂ is measured by ratio comparison to the reference and/or standard gases, like Chromatography technique but it also has a limited to a measurement frequency of one sample every few minutes [1].

Some of the global warming gases that cause to vary the atmospheric temperature with the radiation incoming from outer are CO₂, CH₄, N₂O, SF₆, CO, CFCs, and HCFCs, etc. mention by National Oceanic and Atmospheric Administration, World Meteorological Organization, Global Atmosphere Watch, Central Calibration Laboratory, and they also used to calibrate compressed gas uses these gases [2], [3].

Global warming or Climate change refers to long-term fluctuations in temperature, precipitation, wind, and other elements of the Earth's climate system. Natural processes such as solar-irradiance variations, variations in the Earth's orbital parameters, and volcanic activity can produce variations in climate. According to NOAA report 2020, Tropospheric ozone is formed by two precursor pollutants, volatile organic compounds (VOCs), and nitrogen oxides (NOx), in the presence of ultraviolet light (sunlight).

A. Atmospheric Aerosol and Its Role for Varying Temperature of the Atmosphere

Atmospheric aerosol particles in urban areas, because of the loss of visibility and health effects due to industrialized. This also growth and change of atmospheric concentration aerosols, aerosol dynamics, atmospheric chemistry, meteorology, etc. Water, Ammonia, Sulphuric acid, Vapors can explain atmospheric nucleation with an aerosol of ~1 nm to 3 nm particles.

The formation and growth of atmospheric aerosols and cloud droplets, and their thermodynamic properties help to select the particles either it plays an important for temperature on the variation of atmospheric or not. Since in the atmosphere there are multicomponent, multiphase mixtures, their thermodynamic state, and phase diagrams are typically very complex. Therefore, to obtain thermodynamically consistent vapor pressures, chemical activities, surface tensions, and densities for organic and inorganic compounds and their water solutions as a function of temperature and composition help to participate for atmospheric aerosol [4]-[6]. Vehicular emissions, wood-burning fires, and industrial processes pollute the air and have adverse health effects. Aerosols can be transported from areas of high emissions to relatively clean remote regions, which also have the potential to change the concentration and chemistry in the troposphere and stratosphere. The air pollutant is found for Lumbini is 18 minimum to 209 maximum, for Dang is 26 minimum to 119 maximum, for Chitwan is 42 minimum to 176 maximum, for Nepalgunj 21 minimum to 178, and for Shurkhate 73 minimum to 119 maximum. The air quality present between 2016 to now 2020 is not quite bad but unhealthy and

moderate. These air size help to increasing the atmospheric temperature after collision with aircraft [7].

B. Aircraft Machine Fly Layer and Air Composition

An aircraft is a vehicle that can fly with support of air and counters gravity force by using either static lift or dynamic lift or jet engines. Supersonic and subsonic aircraft fly in the upper troposphere and lower stratosphere that have quite different dynamical and chemical regimes. Supersonic aircraft will fly in the stratosphere near 20 km, where stratospheric residence times due to exchange with the troposphere increase to years. Airplanes fly in the stratosphere due to the least amount of turbulence, very dry, fewer clouds, much smoother, etc. The increases in NO_x typically lead to increases in ozone; ozone changes depend on the complex coupling among HO_x , NO_x , and halogen reactions more detail in Wahner et al., and NOAA.

Our goal is to develop and study the effect of the atmospheric temperature variation with for the collision of small and big particles like aircraft machine and major air composed, aerosols particles, another constituent of the atmosphere, etc. to develop and study the relation of model, we consider the classical theory of collision between two particles.

C. Collision

A collision is an event in which two or more bodies exert forces on each other for a relatively short time. During the collision, each object feels a force for a short period, an impulse, or changes the momentum, velocity. Since there are two types of collision or scattering either in the quantum or classical physics and they are elastic and inelastic which depend upon the law of conservation of momentum and energy before and after the collision.

1) Elastic Collision

In the elastic collision total momentum and the total energy before and after the collision or scattering are conserved. Consider two masses, m_1 and m_2 moving with speed u_1 and u_2 , before the collision and after the collision of these masses velocities is v_1 and v_2 , therefore from the law of conservation of momentum:

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad (7)$$

The conservation of Kinetic Energy says:

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 \quad (8)$$

On solving (7) and (8), and arranging,

$$u_1 - u_2 = v_2 - v_1 \quad (9)$$

This shows that the velocity of the approach is equal to the velocity of separation that is the velocity before the collision and after the collision difference is the same. To determine any of the velocity before and after the collision from (9):

$$v_2 = u_1 - u_2 + v_1 \quad (10)$$

On substituting the value of v_2 in (7) and solving for

$$v_1 = \frac{(m_1 - m_2)u_1 + 2m_2 u_2}{(m_1 + m_2)} \quad (11)$$

Similarly,

$$v_2 = \frac{(m_2 - m_1)u_2 + 2m_1 u_1}{(m_1 + m_2)} \quad (12)$$

2) Cases for our study during an elastic collision

When two bodies are of equal masses suffer elastic collision, then after the collision, they will interchange their velocities, that is $v_1 = u_2$ and $v_2 = u_1$. This is not our case of study because in our consideration the focusing on heavy mass like a rocket, aircraft, etc. and another lighter like hydrogen, oxygen, nitrogen, etc., for a similar reason one cannot consider the case of masses that are $m_1 = m_2 = m$ and the second body is initially at rest i.e., $u_2 = 0$. In this case, $v_1 = 0$ and $v_2 = u_1$. This indicates that after the collision the striking body is stopped in its path and the target body takes off with the initial velocity.

Some of the speed of aircraft (passenger airplane) and speed of major air molecules are listed by skycomp in 2020 are tabulated in table I, below.

TABLE I: VELOCITY OF AIRCRAFT AND AIR MOLECULES

Aircraft	Speed (m/sec)	Air Molecules	Speed (m/sec) at 298k
Boeing 787 Dreamliner	265.55	Oxygen	481.94
Dassault Falcon 900LX	298.33	Nitrogen	515.03
Bombardier Global 6000:	303.88	Argon	431.34
Dassault Falcon 7X	305.55	Carbon dioxide	411.07
Airbus A380	301.94	Methane	681.57
Tupolev Tu-144	675	Hydrogen	1927.78
Cessna Citation X+	320.27	Sulphur dioxide	340.60

This table both air molecules and airspace mechanics has mixed velocities i.e., in some case aircraft have high velocities while in other case air molecules have high velocity.

Case 1: Considering m_1 is mass of aircraft and u_1 is the velocity of aircraft before the collision, for the case of $m_1 > m_2$ and $u_1 > u_2$, then the final velocities of aircraft from equation (11) is written as:

$$v_1 = \frac{m_1 u_1}{(m_1 + m_2)} \quad (13)$$

On neglecting the $2m_2 u_2 \ll m_1 u_1$, and after arranging,

$$v_1 = \frac{u_1}{\left(1 + \frac{m_2}{m_1}\right)} \quad (14)$$

Since $m_1 \gg m_2$ therefore the ratio of $\frac{m_2}{m_1} \ll 1$ so from (14):

$$v_1 = u_1 \quad (15)$$

This shows that before the collision and after the collision the speed of the aircraft is the same, which is a noteworthy case of our study.

Case 2: Considering m_1 is mass of aircraft and u_1 is the velocity of aircraft before the collision, in this case also $m_1 > m_2$ and $u_2 > u_1$, then the final colliding velocities of air molecules or aerosol or similar sized particles with aircraft from equation (12) is written as:

$$v_2 = \frac{-m_1 u_2 + 2m_1 u_1}{(m_1 + m_2)} \quad (16)$$

$$v_2 = \frac{m_1(2u_1 - u_2)}{(m_1 + m_2)}$$

$$v_2 = \frac{(2u_1 - u_2)}{\left(1 + \frac{m_2}{m_1}\right)} \quad (17)$$

Since $m_1 \gg m_2$ therefore the ratio of $\frac{m_2}{m_1} \ll 1$ so from (17):

$$v_2 = 2u_1 - u_2 \quad (18a)$$

$$v_2 = 2v_1 - u_2 \quad (18b)$$

[from equation (15)]

This equation shows that the final velocity of air molecules after collision with aircraft is the subtraction of twice of initial velocity or final velocity of aircraft from an initial velocity of air molecules.

Since at a time a large number of collisions take place with the airplane, therefore, (18) can be written as the sum of all velocities of molecules that collide with the airplane and the equation can be modified as:

$$v_2 = \sum_{i=1}^N \frac{2u_1 - u_i}{\left(1 + \frac{m_i}{m_1}\right)} \quad (19)$$

Also, from (18a) and (18b),

$$u_{rms} = v_2 = 2u_1 - u_2 = \sqrt{\frac{3KT}{m_2}} \quad (20a)$$

$$u_{rms} = v_2 = 2v_1 - u_2 = \sqrt{\frac{3KT}{m_2}} \quad (20b)$$

This relation did not specify the relation either temperature decrease or increases after the collision because the variation of the speed plays an important role. For example, let us consider from the table above Sulphur oxide molecules with $u_2 = 340.60$ m/s collide with aircraft velocity $u_1 = 675$ m/s, then from (20a) or (20b):

$$v_2 = 2 \times 675 - 340.60 = \sqrt{\frac{3KT}{m_2}} \quad (21)$$

$$v_2 = \frac{1,009.4m}{sec} = \sqrt{\frac{3KT}{m_2}}$$

Which is greater than u_2 , so the temperature increases in this case. On considering another example from the above table Sulphur oxide molecules with $u_2 = 340.60$ m/s collide with aircraft velocity $u_1 = 320.27$ m/s, then from (20a) or (20b),

$$v_2 = 2 \times 320.27 - 340.60 = \sqrt{\frac{3KT}{m_2}} \quad (22)$$

$$v_2 = \frac{300.54m}{sec} = \sqrt{\frac{3KT}{m_2}}$$

Which is less than u_2 so the temperature decreases in this case. This shows that temperature increase if the collision of gases take place with u_1 or $v_1 > u_2$, and decrease if $u_2 > u_1$ or v_1 .

III. RESULT AND DISCUSSION

On considering the collision of the particle for our work, and applied the suitable condition fitted to the collision rules. In this work, the development of the relationship of the temperature before and after the collision of air molecules and aircraft machines. Since authors are trying to study the variation of temperature due to the collision of air molecules and aircraft machines, here authors are not considering other collisions with occur in the atmosphere in which one can say the limitation of this study.

The derived relation and examples from (13) to (22), help us to understand the variation of temperature during the collision. But in general, one can also develop a relation (20a), for considering the variation of temperature i.e., increase after the collision, $u_1 > u_2$, therefore (20a), can be written as:

$$v_2 > 2u_1 - u_2 = \sqrt{\frac{3KT}{m_2}} \quad (24)$$

This relation shows that temperature is proportional to velocity as $v_2 \propto \sqrt{T}$ as the other is constant. In this way, authors can represent the increase in temperature of air molecules which is the constituent and critical parameters for atmospheric temperature, because after the collision the velocity goes increase.

Also, for the variation of temperature i.e., decreases after the collision, $u_2 > u_1$ therefore (20a), can be written as:

$$v_2 < 2u_1 - u_2 = \sqrt{\frac{3KT}{m_2}} \quad (25)$$

This relation shows that temperature is proportional to velocity as $v_2 \propto \sqrt{T}$ as the other is constant. In this way, authors can represent the decrease in temperature of air molecules which is the constitute and critical parameters for atmospheric temperature, because after collision velocity goes decrease.

These two equations (24) and (25) clearly, show the effect the variation of temperature, when air molecules colloid with aircraft machine which takes flight in atmospheric air within air density 1.1 to 1.4 kg m⁻³. This variation study of temperature also helps to understand the other collision take place in the atmosphere.

IV. CONCLUSION

The developed relation above for the collision of air molecules and aircraft machines for variation of temperature in the atmosphere. The developed relation shows that when air molecules collide with aircraft, there is no change in the velocity of aircraft machines shown in (15) but there is a change in velocity of air molecules shown in (20). These change velocities after the collision with aircraft indicate the variation of the temperature because the velocity and temperature are related to each other.

Hence in this way, the temperature variation takes place in the atmosphere due to the collision of air molecules with aircraft machines or other machines take the flight in it.

DECLARATION STATEMENT

All authors are equally contributed to this work.

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