Comments on the Black Hole War

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ABSTRACT

We first explain various fundamental concepts. Next, following Susskind’s book, we review the black hole war, which is the 20 years of discussion between Hawking and Susskind, ’t Hooft on the issue of the vanishing of information by the black hole. Finally, we give some questions and comments. We clearly distinguish between the original concept and its analogy.

Keywords: Action at a distance, Hawking effect, Holographic principle, Quantum entanglement.

1. Introduction

The vanishing of the information by the black hole is the important issue nowadays. In order to overcome that issue, we first explain various fundamental concepts. Next, following the Susskind’s book [1], we review the black hole war, which is the 20 years of discussion between Hawking and Susskind, ’t Hooft on the issue of the loss of information by the black hole. Finally, we give some questions and comments.

Not only for amateurs but also for experts, we frequently use the analogy instead of the original concept in order to explain the high-level contents to all people. But we physicist must clearly distinguish between the original concept and it’s analogy. This paper is based on the talk of Nara SA seminar on December 9, 2023 [2].

2. Various Fundamental Concepts

Here we explain the fundamental concept to understand the issue of the vanishing of the information by the black hole.

2.1. Black Hole

2.1.1. Classical Black Hole

In classical mechanics, the condition of the star becomes the black hole is given by:

\[ \frac{\text{second cosmic velocity}}{\text{velocity to escape from the star}} = \text{speed of light}, \]

which gives:

\[ \frac{1}{2}mv^2 - G \frac{Mm}{r} = 0 \Rightarrow \frac{1}{2}mc^2 - G \frac{Mm}{r} = 0 \]

which gives the radius of the event horizon:

\[ r_H = \frac{2GM}{c^2} \]

We classify black holes according to the density of stars [3].
If we assume that the density $\rho$ of stars is constant, by using $M = \rho \times 4\pi r^3/3$, if $r \to \text{large}$, the star always becomes the black hole at some large $r$:

$$\frac{1}{2}mc^2 - G\frac{4\pi r_H^2\rho m}{3} \leq 0$$

2.1.2. Various Types of Black Holes
2.1.2.1. Hydrogen (Density of the Lightest Atom) Type Black Hole
Michel (1784): $r_H = 500R_\odot \Rightarrow$ supermassive black hole at the center of various galaxies. Example, at the center of our galaxy Sgr A*: $M = 4 \times 10^6 M_\odot$ and at the center of $M87$: $M = 6.5 \times 10^9 M_\odot$.

2.1.2.2. Neutron (Density of the Nuclei) Type Black Hole
$r_H = 120$ km $\Rightarrow$ fixed star type black hole. Example: Cygnus X-1; $M = 40 M_\odot$.

2.2. Schwarzschild Metric: (Black Hole Metric)

$$ds^2 = -\left(1 - \frac{2GM/c^2}{r}\right)c^2(dt)^2 + \frac{(dr)^2}{1 - \frac{2GM/c^2}{r}} + r^2(d\theta)^2 + \sin^2(\theta)(d\phi)^2$$

The radius of the event horizon (radius where something irregular happens) is given by $r_H = 2GM/c^2$, which gives the same result as that in the classical theory.

Then we obtain $a = \text{(surface acceleration)} = \text{(acceleration at the event horizon)} = GM/r_H^2 = c^2/4GM$ from $F = ma = GMm/r^2$, and the surface acceleration becomes inversely proportional to black hole mass $M$. This is the reason why the Hawking temperature is inversely proportional to the black hole mass $M$.

2.3. Isotropic Coordinate: More Meaningful Metric

In the Schwarzschild metric, the role of timelike and spacelike direction of $cdt$ and $dr$ is exchanged when we consider metric outside or inside of the event horizon. While, $r\sin\theta d\phi$ and $r\sin\theta d\phi$ directions remain to be spacelike. Then it is more desirable to use the more meaningful metric, the isotropic coordinate [4], [5], to analyze the free fall motion of the matter into the black hole, which is given in the form:

$$ds^2 = -\left(1 - \frac{GM/2c^2}{R}\right)^2(dt)^2 + \left(1 + \frac{GM/2c^2}{R}\right)^4(dX)^2 + dY^2 + dZ^2$$

$$r = R\left(1 + \frac{GM/2c^2}{R}\right)^2, \quad R^2 = X^2 + Y^2 + Z^2, \quad R_H = \frac{GM}{2c^2} \left(= \frac{1}{4r_H}\right)$$

In this isotropic metric, $g_{\mu\nu}$ have no singularity except $R = 0$, but $g^{\mu\nu}$ have the singularity on the event horizon as we obtain $\det g_{\mu\nu}|_{R=R_H} = 0$. Then we cannot take the local inertial frame and the geodesic equation becomes singular on the event horizon.

3. Various Entropies

3.1. Introducing the Entropy

Through the study of the Carnot cycle, Clausius introduce the concept of the entropy (1865). In order to examine whether the heat engine is the most appropriate one, Clausius introduce the entropy in the form:

$$dS_{\text{(idealistic)}} = \frac{dQ}{T}, \quad (S: \text{entropy}, \quad Q: \text{heat}, \quad T: \text{absolute temperature})$$

In the Carnot cycle, isothermal expansion (absorb heat, increase the entropy) $\Rightarrow$ adiabatic expansion (entropy does not change) $\Rightarrow$ isothermal compression (radiate heat, decrease the entropy) $\Rightarrow$ adiabatic compression (entropy does not change) and return to the starting pressure and the volume. In this cycle, we have:

$$\int dS = \oint \frac{dQ}{T} = 0$$

and the change of the entropy becomes just zero. The meaning of this relation is that this cycle is the most appropriate cycle and is reversible. Especially, in order to make the thermal process of the cycle to be reversible, the thermal process in the cycle must be quasi-static, which keeps the state in equilibrium.
3.2. Original Meaning of Entropy

For the system with, (i) many particles system (the number of particles is in the order of Avogadro number), (ii) in thermal equilibrium with temperature $T$, (iii) closed system with finite volume $V$, the entropy is the quantity to measure whether each thermal process is reversible or not, especially thermal process is quasi-static or not.

3.3. Clausius Entropy

Clausius introduced the entropy as the principle to examine whether the heat engine is reversible or not in the form $[6]$:

$$ dS(\text{actual}) \geq dS(\text{idealistic}) = \frac{dQ}{T} $$

3.4. Boltzmann Entropy

Boltzmann introduced the entropy as the object of the number of states in the microcanonical ensemble $[7]$. From the thermal equilibrium state with temperature $T$, volume $V$, we define the entropy $S$ through the number of states $W(E, V)$ with given energy $E$ in the form

$$ S = k_B \log W(E, V) $$

Obtaining the number of states, and replacing the energy $E$ with the temperature by the relation $E = \frac{3}{2}(Nk_BT)$, the entropy is given by $S = (3/2)(nR \log(pV^γ)) + (\text{const.}) = (3/2)(nR \log(TV^γ)) + (\text{const.})$. Because the equation of state for gas molecules $pV = nRT$ is always satisfied for the idealistic case, if the pressure and the volume return to the initial value in the thermal cycle, $\oint dS = 0$ is automatically satisfied.

3.5. Gibbs Entropy

Gibbs introduced the entropy as the object of the probability in the canonical ensemble $[8]$. We naively consider to be (the number of state) $\sim 1/(\text{probability})$ and we consider the Maxwell-Boltzmann distribution in thermal equilibrium as some probability. Then we consider the entropy as the average of the logarithm of the number of the state in the form:

$$ S = \sum_i P(\epsilon_i) \log \frac{1}{P(\epsilon_i)} $$

This Gibbs entropy gives $S = (3/2)(nR \log TV^γ) + (\text{const.})$, which give the same result as that of Boltzmann.

3.6. Shannon Entropy, von Neumann Entropy

By using the Gibbs’s method, even if there are no circumstances of the thermal equilibrium or finite volume, if there is some probability, we can define the analogy of the entropy. Naively, we consider the number of state $\sim \text{information amount} \sim 1/\text{probability}$, and we identify $P(E_i) = \text{probability for the event } E_i$.

a) Selection information amount $= \text{self entropy}$

$$ S = \log_2 \left( \frac{1}{P(E_i)} \right) $$

Probabilistically rare event happens $\Rightarrow$ (many information) $\Rightarrow 1/P(E_i) = \text{(information amount)}$.

Comments: base of log is 2 instead of $e$, and there is no $k_B$ (which has dimension) in front.

b) Average amount of information $= \text{Shannon entropy, von Neumann entropy}$

$$ S = \sum_i P(E_i) \log_2 \left( \frac{1}{P(E_i)} \right) $$

3.7. Bekenstein’s Black Hole Entropy

Bekenstein introduce the analogy of the entropy for the black hole in 1973 $[9]$. Black hole is not in the thermal equilibrium and is not the closed system with the volume being not finite, then the black hole entropy is the analogy of the entropy. Using the analogy of $dU = TdS − pdV$, by putting $U = Mc^2$ (=the rest energy of the black hole), $T_H = (\text{Hawking temperature})$ in the form $dU = T_H dS$, we obtain that the black hole entropy $S$ is proportional to the area of the event horizon. (We will show later.)

Later, we derive the Hawking temperature $[10]$, but we here give the rough estimate of the Hawking temperature and give the rough estimate of the black hole entropy.

**Hawking temperature:** ($M = 6M_\odot$, $T_H = 10^{-8}K$)
\[ k_B T_H = \hbar \frac{c}{\lambda_H} = \frac{\hbar c^3}{8\pi GM} = \frac{\hbar}{2\pi} \frac{a}{c} \]

\[ (\lambda_H = 8\pi^2 r_H \sim r_H = \text{wave length of the order of the event horizon}) \]

If the matter rotates around the event horizon of the black hole, the frequency of rotation is given by \( \nu_H = \frac{c}{2\pi r_H} = c^3/4\pi GM \). By using this frequency, we can estimate the Hawking temperature as the quantum energy for this rotating mode in the form \( k_B T_H = (1/4\pi)(\hbar \nu_H) \), which gives the Hawking’s result except the constant factor \( 1/4\pi \).

Next, we estimate the black hole entropy in the form:

**Black hole entropy:**

\[ G = h = c = 1, \ M \sim r_H, \ T_H \sim 1/r_H, \ dM = TdS \rightarrow S \sim r_H^2 \sim M^2 \]

Exactly, \( S = \frac{k_B c^3}{4G\hbar} \times 4\pi r_H^2 = \frac{k_B}{4} \frac{4\pi r_H^2}{\hbar} \),

\[ (r_H = \frac{2GM}{c^2}, \, \ell_P = \sqrt{\frac{\hbar G}{c^3}} = \text{Planck length} = 1.6 \times 10^{-35} \text{ m}) \]

4. **Quantum Entanglement**

Original meaning of the quantum entanglement is the correlation at a distance in quantum theory, which is usually understand as the creepy phenomena which occurs only for the quantum system. But *the quantum entanglement is just only the conservation law in quantum system*. If we take the sum of the angular momentum of all locations at a certain time, the total angular momentum is conserved. Such conserved law looks like that there exists the correlation at a distance, but it is just only the result of the conservation law. The classical and/or quantum entanglement is used as the analogy of (i) the classical and/or quantum conservation law, (ii) the action at a distance. The Coulomb force and/or gravitational force, which is mediated by the non-quantized longitudinal mode, causes the action at a distance [11], and they are really physical. We can obtain the Coulomb force from \( \mathbf{F} = q\mathbf{E} \) and \( \mathbf{E} = \mathbf{A} \), that is, it is not necessary to use potentials \( \phi \) and \( \mathbf{A} \), so that it is not necessary to fix the gauge. From inside the black hole, any kind of particle (quantized transverse mode) cannot come out, but the gravitational force (non-quantized longitudinal mode) comes out from inside of the event horizon of the black hole.

**Example:** For the rotational motion of binary stars interacting with the gravitational force, the total angular momentum is conserved only when the gravitational force acts at a distance. In this example, the conservation law and the action at a distance is strongly related.

5. **Hawking Effect**

5.1. **Equivalence Principle of the Gravitational Force**

We cannot distinguish between gravitational force and acceleration:

1) frame without gravitational force: free fall frame, local inertial frame = frame without acceleration: inertial frame

**Example:** frame in the free fall elevator = inertial frame

2) frame of the gravitational force = frame of the acceleration

**Example:** frame of the gravitational force on earth = frame of acceleration in the rocket at outer space

We connect the local inertial frame (free fall frame) with the frame of the gravitational force by the general coordinate transformation. From the relation of bases of such frames, we can connect the creation and annihilation operators between two frames. In such a situation, the observer at the local inertial frame observes the phenomena of the creation of the particle in the vacuum at the frame of the gravitational force. (Bogoliubov transformation) We will later discuss the detailed meaning of this phenomenon.

We use the local coordinate \( \rho \), which has dimension of length, defined in the form:

\[ r = \frac{2GM}{c^2} + \rho^2 \frac{c^2}{8GM} \]

By using this local coordinate, the metric is given by:

\[ (ds)^2 = -\rho^2 \left( \frac{c^3}{4GM} \right)^2 + (d\rho)^2 = -\rho^2 \left( \frac{a}{c} \frac{a}{c} \right)^2 + (d\rho)^2 = -\rho^2 d\hat{t}^2 + (d\hat{t})^2 \]
where, \( \hat{t} = (a(\text{surface})/c)(t) \) and \( \hat{t} \) is the dimensionless quantity. Because the above metric is that of the constantly accelerating frame, we use the Rindler metric, which connect the frame of the flat space-time with the frame of the constantly accelerating space-time. Here, we consider only two directions of \( dt \) and \( dx \), and we obtain:

\[
(d\hat{t})^2 = -(dt)^2 + (dx)^2 = -\hat{z}^2(\hat{\eta})^2 + (\hat{\xi})^2
\]

where, relations between various coordinates are given by \( \hat{t} = z \cosh \eta \), \( x = z \sinh \eta \), \( \rho = z \), \( \eta = \hat{t} \), and we further put \( z = \hat{e}^\xi \). Then \( (t, x) \) and \( (\eta, \xi) \) are related by the relation \( t = \hat{e}^\xi \sinh \eta \), \( x = \hat{e}^\xi \cosh \eta \). Only for simplicity, we consider the scalar field and expand this scalar field both in the flat space-time and in the constantly accelerating space-time in the form:

\[
\phi = \int \frac{d\omega}{\sqrt{2|\omega|}} \left[ a(\omega)e^{-i(t-x)\omega} + a^\dagger(\omega)e^{i(t-x)\omega} \right] = \int \frac{d\omega'}{\sqrt{2|\omega'|}} \left[ b(\omega')e^{-i(t-\xi)\omega'} + b^\dagger(\omega')e^{i(t-\xi)\omega'} \right]
\]

By using the relation of bases of both expansions, we can connect the creation and the annihilation operators in both space-time, which gives:

\[
b(\omega') = \int d\omega[a(\omega, \omega')a(\omega) - \beta(\omega, \omega')\alpha(\omega)], \quad b^\dagger(\omega') = \int d\omega[a^\dagger(\omega, \omega')a(\omega) - \beta^*(\omega, \omega')\alpha(\omega)]
\]

Coefficients \( \alpha, \beta \) satisfies the following relation:

\[
\int d\omega \left[ \alpha(\omega, \omega')\alpha(\omega, \omega'') - \beta(\omega, \omega')\beta(\omega, \omega'') \right] = \delta(\omega' - \omega'')
\]

The vacuum state \( |0 \rangle \) in the flat space-time satisfies \( a|0 \rangle = 0 \), but it becomes \( b|0 \rangle \neq 0 \). Then the density of the number of the particle at the constantly accelerating space-time becomes non-zero in the form:

\[
< 0|b^\dagger(\omega')b(\omega')|0 > = \int d\omega|\beta(\omega, \omega')|^2
\]

Thus, the observer at the frame of the flat space-time observes the creation of the particle at the frame of constantly acceleration space-time, and the density of the number of created particles is given by:

\[
< 0|b^\dagger(\omega')b(\omega')|0 > = \int d\omega|\beta(\omega, \omega')|^2 = \frac{1}{e^{2\pi\omega} - 1}
\]

Finally, by noticing the relation, \( \omega' = \omega'(a(\text{surface})/c)t = \hat{\omega}t \), we express the density of the number of particles in the form:

\[
\frac{1}{e^{2\pi\omega} - 1} = \frac{1}{e^{\omega/\kappa_B T_H} - 1}
\]

Thus, we obtain the Hawking temperature \( k_B T_H \) in the form [10]:

\[
k_B T_H = \frac{\hbar}{2\pi}\frac{a(\text{surface})}{c} = \frac{\hbar c^3}{8\pi GM}
\]

By using the analogy of the thermodynamics relation \( dU = TdlS \), and we make the correspondence between the thermodynamics quantities and the black hole quantities as follows: \( U \leftrightarrow M e^2 = \) (rest energy of the black hole), \( T \leftrightarrow T_H = \) (Hawking temperature), and \( S \leftrightarrow S_B = \) (black hole entropy). Therefore, we obtain the entropy of the black hole \( S_B \) in the form:

\[
S_B = \frac{k_B M e^2}{4\pi G h c} \times M^2
\]

By using the relation \( r_H = 2GM/c^2 \), rewriting \( M \) with \( r_H \), we obtain the expression:

\[
S_B = \frac{k_B c^3}{4Gh} \times 4\pi r_H^2 = \frac{k_B 4\pi r_H^2}{4 \ell^2_P} = \frac{k_B (\text{area of the surface of the event horizon})}{\ell^2_P}
\]

and we obtain the area law of the black hole entropy.
temperature given by Hawking. The singularity of the space-time with

\[ -t^2 - r_0^2 + x^2 + y^2 + z^2 + w^2 = -\ell^2 \]

= 1/\Lambda (metric on the surface)

Black hole with cosmological term \( \Lambda \)

5.2. Susskind’s Estimate of the Black Hole Entropy

Susskind intuitively estimated the black hole entropy in the following way [1]. If the photon, whose wave length is in the order of the radius of the event horizon, falls into the black hole, we consider that one unit of information falls into the black hole. Then the change of mass of the black hole is given by \( \Delta E \sim h\nu = hc/\lambda \), and we obtain \( \Delta (M c^2) = (1/8\pi^2)(hc/r_H) \) except the constant factor \( 1/8\pi^2 \). By using \( r_H = 2GM/c^2 \), we obtain \( \Delta (4\pi r_H^2) = ((4\tilde{G}h)/c^3) = 4\ell_p^2 \), and we obtain that the information entropy increased by the amount of \( \Delta (4\pi r_H^2)/4\ell_p^2 = 1 \) bit. By multiplying \( k_B \) to the information entropy, the increase of the black hole entropy is given by \( \Delta S = k_B((\Delta (4\pi r_H^2))/4\ell_p^2) \), which gives the correct black hole entropy.

Comment: Can we apply the quantum theory to the cosmologically large black hole? Is it not absurd just like to use the Schrödinger equation for the system of sun and earth interaction with gravitational potential and consider the probability to exist the sun?

If we clarify the situation and consider the meaning of the black hole effect, we come to the paradoxical phenomena, which is well known from old days. The paradoxical phenomena are the followings: The observer in the frame of free fall observes the radiation of light from the charged particle in the frame of the gravitational force. But the observer in the frame of gravitational force does not observe the radiation of light from the charged particle in the frame of the gravitational force [12].

In the Hawking effect, the observer in the frame of free fall observes the radiation of light at the vacuum in the frame of gravitational force. While the observer in the frame of gravitational force does not observe the radiation of light at the vacuum in the frame of gravitational force.

6. Holographic Principle

Holographic principle means that the information of the 3-dim. volume is embedded on the surface of 2-dim. surface. ‘t Hooft’s holographic principle [13] seems to be just rewriting the black hole entropy, which is originally proportional to \( M^2 \), into \( 4\pi r_H^2 \) by the dimensional analysis.

\[
S = \frac{k_B 4\pi G}{hc} \times M^2 = \frac{k_B}{4} \frac{\text{(area of the surface of the event horizon)}}{\ell_p^2}
\]

In some sense, we can consider the holographic principle as the Gauss’ theorem. By the Gauss’ theorem, we can know the 3-dim. information (mass, angular momentum, charge etc. of the black hole) on the 2-dim. surface which is located outside of the event horizon. To know the information of the physical object and to get the physical object is totally different.

7. Maldacena’s \( AdS_5/CFT_4 \) Correspondence

We first give the correspondence between \( AdS_5 = (4 + 1)\text{-dim. anti-de Sitter gravitational theory} \) and \( CFT_4 = (3 + 1)\text{-dim. conformal field theory} \) in Table I [14].

\( AdS_5 \) theory is the dimensionful theory because the cosmological term \( \Lambda \) has dimension. While \( CFT_4 \) is the dimensionless theory because it is the massless theory.

Comment 1: Independent of the gravitational theory, the space-time of the hypercone, which is the space-time with \( \ell = 0 \) in \( AdS_5 \) (hypercone), is used to explain the conformal invariance of the \( (3 + 1)\text{-dim. massless field theory from old days (Dirac’s paper in 1936 [15] is the first one to investigate.)} \)

Comment 2: Only for the Schwarzschild type black hole, we obtain the expression of the Hawking temperature given by Hawking. The singularity of the \( AdS_5 \) black hole is different from that of the Schwarzschild black hole, so that it is not guaranteed to obtain the same expression of the Hawking temperature for the \( AdS_5 \) black hole. Then it is not guaranteed that the black hole entropy is expressed as the area law.
7.1. Is There Field Correspondence between AdS$_3$ and CFT$_4$?

Though AdS$_3$ and CFT$_4$ give the same number of symmetries, that is, both theories have 15 symmetries, both symmetry groups are similar but not exactly the same. How to make the one-to-one correspondence between fields of AdS$_3$ and fields of CFT$_4$ in different dimension?

7.2. AdS$_3$/CFT$_4$ Correspondence (Brown-Henneaux)

The origin of AdS$_3$/CFT$_4$ correspondence is the AdS$_3$/CFT$_2$ correspondence by Brown-Henneaux [16]. In AdS$_3$ theory, there exist BTZ “black hole” solution. If we rewrite this (2 + 1)-dim. gravitational theory by using drei-bein field, we obtain the Chern-Simons theory and the action becomes the total derivative. Then the equation of motion is trivially satisfied and we obtain the (1 + 1)-dim. massless conformal field theory. In this example, we make the one-to-one correspondence between (2 + 1)-dim gravitational fields and the (1 + 1)-dim. conformal filed theory.

Is this correspondence really correct?

What happens to the BTZ black hole solution [17], which is the non-trivial solution in (2 + 1)-dim. AdS$_3$ gravitational theory, in the (1 + 1)-dim. conformal field theory? In the drei-bein formalism of AdS$_3$ gravitational theory, we must put the covariant derivative of the drei-bein to be zero in the form $\nabla_{\mu} e^\mu_i = 0$, and we must strongly solve spin connections $\omega^{ab} _i$ and gravitational fields $g_{\mu \nu}$ by drei-beins and their derivatives. While Brown-Henneaux treated dre-beins and spin connections independently.

7.3. Comparison between BTZ “Black Hole” Solution (Anti-de Sitter + Rotation) and AdS$_3$ Solution

If we compare the non-rotating ($J = 0$) BTZ “black hole” solution with the AdS$_3$ solution [18] in the following way:

$$\text{BTZ solution: } ds^2 = -\left( -M + \frac{r^2}{\ell^2} + \frac{J^2}{4\ell^2} \right) \left( dt - \frac{1}{(-M + r^2/\ell^2 + J^2/4\ell^2)} dr \right)^2 + \frac{r^2}{(2\ell^2 dt)} d\phi^2$$

$$\text{AdS}_3 \text{ solution: } ds^2 = -(1 + \frac{r^2}{\ell^2})(dt)^2 + \frac{(dr)^2}{(1 + \frac{r^2}{\ell^2})} + r^2(d\phi)^2$$

we must take $M = -1$. Because, if we take the further limit of $\ell \rightarrow \infty$, the cosmological terms $\Lambda = -1/\ell^2$ becomes zero and the Einstein equation becomes that of the vacuum. Then the metric should become the flat Minkowski space-time. For this AdS$_3$ solution, there is no singularity in the metric so that it is not the “black hole” solution.

8. The Vanishing of Information by the Black Hole: 20 Years of Discussion between Hawking and Susskind, ’t Hooft

Hawking: Cambridge Univ., Lucasian Professorship (Newton, Babbage, Stokes, Dirac): The genius in the wheelchair. He proposes unique ideas: “Time becomes reversed when the Universe changes into the contraction”, “Before the big bang, time begins with imaginary value”, Time machine.

Susskind: Stanford Univ., famous univ. in the west coast of U.S.A. From plumber to elementary particle physicist. By using the analogy, he is good at explaining physics intuitively.

’t Hooft: Utrecht University, Holland. Great talent, Nobel prize winner.

Hawking’s claim:

From inside the black hole, the energy come out by the Hawking mechanism. Then the energy inside the black hole decreases, which means that the matter evaporate and finally vanishes. This is the evaporation and the vanishing of the black hole, so that the information of the matter inside the black hole is finally vanished by the Hawking mechanism.

Susskind’s claim:

For the observer at infinity, he does not observe that the matter falls inside the event horizon. While for the observer, who is freely falling inside the event horizon with the matter, as we can take the local inertial frame, he does not notice the event horizon so that he and the matter falls inside the event horizon. Susskind’s complementary principle means that the observation at infinity and the observation near the event horizon are both true. For the observer at infinity, the matter remains just outside of the event horizon, which is eventually radiated by the thermodynamical pressure with
Hawking temperature, that is, the matter does not fall inside the black hole for the observer at infinity. While the matter falls inside the black hole for the freely falling observer with the matter. Observations for different observers are both true, that is, the matter inside of the black hole is “projected” to the matter outside of the black hole. This is the correspondence between the matter inside the black hole and the matter outside of the black hole through the Susskind's complementary principle. This means that the information of the matter inside the black hole is projected to the information of the matter outside of the black hole.

't Hooft's claim:

The information of the matter inside of the black hole is projected to the surface outside of the black hole.

8.1. Irreversible Process in the Black Hole

Analog of the increase of the entropy and the increase of the black hole mass

Entropy: the entropy does not change or increase but not decrease ⇒ if the entropy increases, the irreversible process occurred.

The mass of the black hole: matter falls into the black hole, but the matter cannot come out from the black hole ⇒ the mass of the black hole does not change or increase but not decrease ⇒ matter falls into the black hole through the irreversible process. What happens if time is reversed? Actual irreversible process occurs through accretion disk. The matter can lose the kinetic energy and the angular momentum by the friction of the accretion disk, and only such matter can fall into the black hole. The acquired kinetic energy and the angular momentum is radiated as a jet from the accretion disk.

8.2. Various Questions and Comments

Q1: What is the meaning that the information instead of the matter falls into the black hole?
Even if the matter falls into the black hole, we can know the information of the matter (mass, angular momentum, charge etc.) from outside of the black hole, that is, the information does not fall into the black hole.

Q2: Does black hole evaporate and finally vanish by the Hawking effect?
By the Hawking effect, even if energy comes out from inside the black hole, the creation of particle occurs outside of the event horizon. We will explain the Hawking effect in the following example. Let's consider that the pair creation occurs outside of the event horizon, and one particle go out of the black hole and the other particle falls into the black hole. In such situation, the falling of the particle inside the black hole causes the increase of the mass of the black hole and the potential energy becomes more negative and the binding becomes stronger. While the outgoing particle cause the radiation of the energy from outside of event horizon of the black hole. The energy comes out from outside of the event horizon of the black hole, but the particle does not come out from inside of the event horizon of the black hole. From the energy conservation, the energy comes out from outside of the event horizon of the black hole and simultaneously the potential energy of the black hole becomes more negative by increasing the black hole mass. Therefore, the black hole does not evaporate and the black hole does not vanish.

Q3: What is the meaning of Susskind's “complementarity principle”? [1]
Susskind consider in the following way: If the observer who fall together with the particle observe that the particle falls inside the event horizon without noticing the existence of the event horizon. While the observer at infinity observes that the particle remains on the event horizon but does not observe that the particle falls inside the event horizon, and the particle eventually is radiated from the place of the event horizon by the pressure of the Hawking radiation, that is, the particle does not fall into the event horizon for the observer at infinity. The Hawking temperature is only the analogy and there is no real pressure of the thermodynamical equilibrium. If the particle does not fall into the black hole, the mass cannot increase but the mass of the supermassive black hole gradually increased from the early stage of the universe. The physical object can have one aspect of properties and simultaneously have another aspect of properties, such as the matter can be the particle and simultaneously can be the wave. But it is impossible to occur that the event occurs for one observer and simultaneously the event does not occur for the other observer. If the observer near the event horizon observes that the particle falls inside the event horizon, such event occurs but did not yet observe that event for the observer at infinity.

9. Summary

We first review fundamental concepts such as the black hole, various entropies, the quantum entanglement, the Hawking effect, the holographic principle, the $AdS_5/CFT_4$ correspondence. Especially, we
pointed out that the quantum entanglement is just only the conservation law in the quantum system, and the action at a distance such as the Coulomb force and the gravitational force is really the physical ones. Then we review claims of Hawking’s, Susskind’s and ‘t Hooft’s on the vanishing of the information at the black hole by the Hawking effect. We give some critical comments on these claims.

CONFLICT OF INTEREST

Author declares that there is no conflict of interest.

REFERENCES