The Effect of Fluctuations in the Observer’s Frame of Reference on Blackbody Radiation

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Abstract — This paper describes how the characteristics of blackbody radiation are affected by the observer’s frame of reference (OFR). To date, the specific intensity of a photon emitted by a blackbody has been studied based on the assumption that the OFR remains constant throughout the performance of measurements of the specific intensity; thus, how much the specific intensity of the photon is affected by fluctuations in the OFR remains unknown. In this paper, the specific intensity of a photon emitted by a blackbody is considered as the OFR fluctuates. The average specific intensity of a photon is formulated for two types of variations in the OFR with time: periodic square-wave and periodic sawtooth fluctuations. For these two types of fluctuations, the average specific intensity of a photon that has a frequency much higher than that corresponding to the amplitude of the changes in the OFR is found to be always lower than for a stationary OFR. It is also found that the average specific intensity is inversely proportional to the temperature in the limit that the temperature is much higher than that corresponding to the amplitude of these changes. The average specific intensity of a photon in a fluctuating OFR could be used to explain the characteristics of the cosmic microwave background radiation as observed by an observer located in the cosmic background.

Keywords — blackbody radiation, cosmic microwave background radiation, fluctuations in the observer’s frame of reference, observer effect, Planck radiation law.

I. INTRODUCTION

Blackbody radiation is the thermal electromagnetic radiation emitted by a blackbody [1] that absorbs all frequencies of radiation in all directions through its surface and emits the same amount of energy as it absorbs when in thermal equilibrium with its environment. Ever since the blackbody radiation law was formulated by Planck in 1900 based on a hypothesis related to the energy quantization of photons [2], this law — the Planck radiation law — has been used as the basis not only for deriving radiation laws such as the Stefan–Boltzmann law [3] and Wien’s displacement law [4] but also for understanding the cosmic microwave background radiation (CMBR) [5–7]. In addition, the Planck radiation law plays an essential role in the search for nonreflective materials such as graphene [8] and for dark matter and dark energy [9], in gravity gradiometry [10], [11], and in gravitational wave detection [12], [13].

Blackbody radiation has been quantitatively studied in terms of the specific intensity of the photons emitted by a blackbody, which is proportional to the number density of the photons multiplied by the average energy of the photons at frequency \( f \). Until now, in studies of blackbody radiation, it has been assumed that the observer’s frame of reference (OFR) remains constant—conventionally, the OFR is assumed to be stationary throughout the performance of measurements of the specific intensity. Thus, how much the average specific intensity of a photon changes for a fluctuating OFR remains an open question.

Recently, a novel observer effect induced by a fluctuating OFR has been proposed for an Einstein solid [14] — a single-electron transistor (SET) [15] — and for tunneling through a one-dimensional (1–D) square potential barrier [16]. Details of the fluctuations in an OFR can potentially be derived from the function describing the molar specific heat at the constant volume of an Einstein solid as a function of temperature. The specific heat has its peak value at low temperatures under periodic square-wave fluctuations, whereas its value converges to three times that of the gas constant at low temperatures under periodic sawtooth fluctuations. It is also possible to derive the transmission probability of a particle from the patterns of the fluctuations in an OFR. For a varying OFR, the average probability of a particle penetrating a square potential barrier monotonically increases with the energy of the particle; this probability reaches saturation to that for a stationary OFR at energy much greater than the amplitude of the fluctuations. The probability rapidly increases just above the energy corresponding to the amplitude of the fluctuations in the OFR in the case of periodic square-wave variations, whereas it gradually increases above this energy in the case of periodic sawtooth fluctuations.

If the reference of energy of a photon is aligned to an OFR, then it is possible to investigate the observer...
effect produced by the fluctuating OFR. In this study, the average specific intensity of a photon emitted by a blackbody was investigated for a fluctuating OFR. To demonstrate the effect of the fluctuations, the average specific intensity was formulated for two types of fluctuations with time: periodic square-wave fluctuations and periodic sawtooth fluctuations. In both cases, the average specific intensity included a fluctuation-induced term, which is a function of the amplitude of the fluctuations divided by the temperature of the blackbody. It is this term that causes the specific intensity to be different from what it would be for a stationary OFR. As mentioned in [16], it is obvious that, in the case of periodic square-wave fluctuations, the average specific intensity also rapidly increases just above the frequency that corresponds to the average specific intensity of the fluctuations divided by the temperature of the blackbody. Under these conditions, the average specific intensity in the case of a varying OFR has the potential to explain the observations of the CMBR obtained from the cosmic background explorer satellite (COBE). The CMBR exhibits small, as yet unexplained, deviations from the specific photon intensity that would be expected for a stationary OFR at frequencies above 400 GHz.

II. BLACKBODY RADIATION

Fig. 1(a) shows a sketch of a system consisting of a blackbody located at the origin \( r = 0 \) and an observer at a distance \( r_1 \); the interface between the OFR and the rest of space is located at a distance \( r_2 \). According to the Planck radiation law [2], the specific intensity (the power radiated per unit surface area of the blackbody opening and per unit solid angle in the frequency range \( f \) to \( f + df \) by a blackbody at temperature \( T \)), \( I_T(f) \), is expressed as:

\[
I_T(f) = \frac{2f^2}{c^2} U_0, \tag{1}
\]

where \( U_0 = hf/\{\exp(hf/k_BT) - 1\} \) is the average energy of a photon in the frequency range \( f \) to \( f + df \). Here, \( c \) is the speed of light, \( h \) is the Planck’s constant and \( k_B \) is the Boltzmann’s constant.

It has been found that \( I_T(f) \) is 0 at \( f = 0 \) and then gradually increases to reach a maximum; subsequently, it decreases with increasing \( f \) and tends to 0. Setting \( dI_T(f)/df = 0 \), the frequency at the maximum specific intensity, \( f_{\text{max},0} \), can be expressed as \( f_{\text{max},0} = 2.821 k_BT/h \), which is known as the Wien’s displacement law [4].

Fig. 1(b) shows a schematic energy diagram of a photon in an OFR, where \( E = hf \) is the energy of a photon and \( E_{\text{OFR}}(t) \) the energy of the OFR at time \( t \). In this study, \( E_{\text{OFR}}(t) \) was assumed to be constant in the time interval \( t \) to \( t + dt \) (\( dt \ll \Delta t (= t_f - t_i) \); \( t_i \) and \( t_f \) are the initial and final times, respectively.) The photon encounters a potential barrier \( E_{\text{OFR}}(t) \). A photon located at a potential barrier with a height \( E_{\text{OFR}}(t) \) \( (r = r_0) \) can be detected by an observer far from the potential barrier \( (r = r_1) \).

For a photon with \( E \geq E_{\text{OFR}} \), in this OFR, the average specific intensity over a time interval \( \Delta t \), \( \bar{I}_T(f) \), can be expressed as:

\[
\bar{I}_T(f) = \frac{1}{\Delta t} \int_{t_i}^{t_f} I_T(f,t)dt = \frac{2f^2}{c^2\Delta t} \int_{t_i}^{t_f} \bar{U}_{\text{OFR}}(f,t) T(f,t) dt \tag{2}
\]

where \( I_T(f,t) \) is the specific intensity at \( t \), \( \bar{U}_{\text{OFR}}(f,t) \) is the average energy of the photon with a frequency \( f \) at time \( t \), which can be expressed as \( \bar{U}_{\text{OFR}}(f,t) = \sum_{n=0}^{\infty} E_n(t) \exp(-E_n(t)/k_BT) / \int_{t_i}^{t_f} \sum_{n=0}^{\infty} \exp(-E_n(t)/k_BT) dt \). Here, \( E_n(t) = E_n - E_{\text{OFR}}(t) = nhf - hf_{\text{OFR}}(t) \) is the measured energy of the photon for \( E_n \) at \( t \). Here, \( T(f,t) \) is the probability that a photon with a frequency \( f \) will penetrate the barrier at time \( t \). For \( f \gg f_{\text{OFR}} \), \( T(f,t) \approx 1 \) [16]. If \( f \gg f_{\text{OFR}} \), \( \bar{I}_T(f) \) can be approximated as:

\[
\bar{I}_T(f) \approx \frac{2f^2}{c^2\Delta t} \int_{t_i}^{t_f} \bar{U}_{\text{OFR}}(f,t) dt. \tag{3}
\]

III. AVERAGE SPECIFIC INTENSITY UNDER PERIODIC SQUARE-WAVE FLUCTUATIONS

For the periodic \( E_{\text{OFR}} \) given by \( E_{\text{OFR}}(t) = \left\{ \begin{array}{ll} \varepsilon_1, & \text{for } 0 \leq t < \tau_1/2 \\ -\varepsilon_1, & \text{for } \tau_1/2 \leq t < \tau_1 \end{array} \right. \) (Fig. 2(a)), \( E_n(t) \), can be expressed as:
where $\epsilon_1$ and $\tau_1$ are the amplitude and period of the periodic square-wave fluctuations, respectively.

A. Single-period Square-wave Fluctuations

For an observer in a frame of reference by means of one-period periodic square-wave fluctuations, the corresponding specific intensity, $I_{T,1}(f)$, can be expressed as:

$$I_{T,1}(f) \approx I_T(f) + \alpha_{T,1}(f) \quad \text{for } f \gg f_1$$

(5)

where $\alpha_{T,1}(f) = -2hf^2f_1\tanh(hf_1/k_BT)/c^2$. Here, $t_1 = 0$, $t_f = \tau_1$ and $f_1 = \epsilon_1/h$.

As shown in Fig. 2(b), at $f_1 = 0$, $I_{T,1}(f)$ is the same as $I_T(f)$. If $f_1 > 0$, $I_{T,1}(f) < I_T(f)$ for all $f \gg f_1$. Setting $dI_{T,1}(f)/df = 0$, the frequency at the maximum intensity, $f_{max,1}$, satisfies the equation of $3f/(\exp(hf/k_BT) - 1) - hf^2/4k_BT\sinh^2(hf/k_BT) = 2f_1\tanh(hf_1/k_BT)$, thus, $f_{max,1}$ gets lower from $f_{max,0}$ as $f_1$ increases. In the high-temperature limit, $k_BT \gg hf_1$, $I_{T,2}(f)$ can be expressed as $I_{T,1}(f) \approx I_T(f) - 2hf^2f_1^2/k_BTc^2$ at $f \gg f_1$.

B. Periodic Square-wave Fluctuations over Long Time Periods

For an observer in a fluctuating frame of reference with periodic square-wave fluctuations, in the long-term limit, the specific intensity, $I_{T,1}'(f)$, can be expressed as:

$$I_{T,1}'(f) \approx I_{T,1}(f)$$

(6)

where $t_1 = 0$ and $t_f \gg \tau_1$.

IV. AVERAGE SPECIFIC INTENSITY UNDER PERIODIC SAWTOOTH FLUCTUATIONS

For the periodic $E_{OFR}$ given by $E_{OFR}(t) = \epsilon_2 - 2\epsilon_2t/\tau_2$ for $0 \leq t < \tau_2$ (Fig. 3(a)), $E_n(t)$ can be expressed as:

$$E_n(t) = E_n - \epsilon_2 + 2\epsilon_2t/\tau_2, \text{ for } 0 \leq t < \tau_2$$

(7)

where $\epsilon_2$ and $\tau_2$ are the amplitude and period of the periodic sawtooth fluctuations, respectively.

A. Single-period Sawtooth Fluctuations

For an observer in a fluctuating frame of reference by means of one-period periodic sawtooth fluctuations, the specific intensity, $I_{T,2}(f)$, can be expressed as:

$$I_{T,2}(f) \approx I_T(f) + \alpha_{T,2}(f) \quad \text{for } f \gg f_2$$

(8)

where $\alpha_{T,2}(f) = 2hf^2(k_BT/h - f_2\coth(hf_2/k_BT))/c^2$. Here, $t_1 = 0$, $t_f = \tau_2$ and $f_2 = \epsilon_2/h$.

As shown in Fig. 3(b), at $f_2 = 0$, $I_{T,2}(f)$ is the same as $I_T(f)$. At $f_2 > 0$, $I_{T,2}(f) < I_T(f)$ for all $f \gg f_2$. In addition, if $f_2 = f_1$, $I_{T,2}(f)$ satisfies $I_{T,1}(f) < I_{T,2}(f) < I_T(f)$ for all $f \gg f_2$.

B. Periodic Sawtooth Fluctuations over Long Time Periods

For an observer in a fluctuating frame of reference with periodic sawtooth fluctuations, in the long-term limit, the specific intensity, $I_{T,2}'(f)$, can be expressed as:

$$I_{T,2}'(f) \approx I_{T,2}(f)$$

(9)

where $t_1 = 0$ and $t_f \gg \tau_2$.

V. RELEVANCE OF THE EFFECT OF OFR FLUCTUATIONS ON BLACKBODY RADIATION TO THE CMBR

A shown in Fig. 4, the COBE mission found that the CMBR corresponds to the radiation emitted by a perfect blackbody [5]–[7]. Fitting $I_T(f)$ (depicted in gray in Fig. 4) to the CMBR data gives the temperature of blackbody $T = 2.725 \pm 0.001$ K, and the fit for the peak of the CMBR is exceptionally good. However, the theoretical values start to deviate from the observations above 400 GHz, and this discrepancy increases with increasing $f$. A correction is therefore needed. As an alternative, $I_{T,1}(f)$ (depicted in black in Fig. 4)
provides a better fit to the CMBR data over all frequencies, giving values of $T = 2.727 \pm 0.001$ K and $\epsilon_1 = 0.547 \pm 0.050$ GHz. $\tilde{I}_{T,2}(f)$ (the dotted black line) also provides a better fit to the CMBR data, giving $T = 2.727 \pm 0.001$ K and $\epsilon_2 = 0.947 \pm 0.087$ GHz. In fact, $\tilde{I}_{T,2}(f)$ is the same as of $\tilde{I}_{T,1}(f)$ if $\epsilon_2 = \sqrt{3} \epsilon_1$, which corresponds to high-frequency and high-temperature regimes, because the amplitude of the OFR fluctuations is then of the order of 10 mK and therefore much smaller than a temperature of about 3 K. Thus, the CMBR data obtained by the COBE mission cannot distinguish between periodic square-wave fluctuations and periodic sawtooth fluctuations in the OFR. Nevertheless, the CMBR data can be better understood if it is considered that the detected blackbody radiation corresponds to the radiation that would be detected for a fluctuating OFR rather than for a stationary one.

Fig. 1. (a) Sketch of a system consisting of a blackbody at $r = 0$ and an observer at $r = r_1$. A photon emitted by the blackbody is located within the observer’s frame of reference (OFR) at $r = r_0$ and detected by the observer at $r = r_1$. (b) Spatial distribution of the energy of the system, where $hf$ is the energy of a photon; $E_{\text{OFR}}(t)$ the reference energy of the photon at time $t$.

Fig. 2. (a) Representation of a frame of reference with periodic square-wave fluctuations, $E_{\text{OFR}}(t)$, for which the amplitude and period are $\epsilon_1$ and $\tau_1$, respectively. The measured energy at time $t$, $E(t) = E - E_{\text{OFR}}(t)$; for a stationary frame of reference, $E_{\text{OFR}}$ is taken to be equal to 0. (b) Average specific intensity (SI) of a photon in the case of periodic square-wave fluctuations as a function of $f$, $\tilde{I}_{T,1}(f)$. The SI is shown for $\epsilon_1 = 0$ GHz (black), 5 GHz (gray), and 10 GHz (light gray).

Fig. 3. (a) Representation of a frame of reference with periodic sawtooth fluctuations, $E_{\text{OFR}}(t)$, for which the amplitude and period are $\epsilon_2$ and $\tau_2$, respectively. (b) Average SI of a photon in the case of periodic sawtooth fluctuations as a function of $f$, $\tilde{I}_{T,2}(f)$. Here, $\epsilon_2 = 0$ GHz (black), 5 GHz (gray), and 10 GHz (light gray).
In this paper, the characteristics of blackbody radiation for an OFR with periodic fluctuations were described, and the average specific intensity of a photon emitted by a blackbody was calculated. Based on the assumption that the reference of energy of the photon was aligned to the OFR, the average specific intensity was formulated for two types of fluctuations with time: periodic square-wave fluctuations and periodic sawtooth fluctuations. For single-period square-wave fluctuations or in the long-term limit for this type of fluctuation, it was shown that the average specific intensity first gradually increases as the photon frequency increases; the specific intensity then reaches a maximum before decreasing as the frequency increases further. As the amplitude of the fluctuations in the OFR increases, the peak value of the intensity decreases. For single-period sawtooth fluctuations or in the long-term limit, it was shown that the average specific intensity varies with the photon frequency in a similar way to the square-wave case. However, at high frequencies, the intensity is always higher than that for the square-wave fluctuations. Therefore, the average specific intensity of a photon provides a means of characterizing the nature of fluctuations in the OFR. At high frequencies and high temperatures, the average specific intensity of a photon emitted by a blackbody is proportional to the amplitude of the fluctuations divided by the temperature of the blackbody. This insight has the potential to be adopted as an explanation of the characteristics of the CBMR obtained by the COBE mission.

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CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

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