The ES Log-normal Distribution Determined by the Einstein Median as the Scale Parameter and the Shannon Shape Parameter

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Abstract — The guiding principle of this contribution is the mutual interplay between the Solar gravitational field and the Maxwell-Boltzmann distribution of speeds of atoms and the observed Fraunhofer lines. We know from numerous experiments that the Newtonian gravitational constant does not depend on the atomic mass, temperature, pressure and many other particle parameters. Therefore, we should discover a universal distribution function that could be used for all atoms and their properties for a given gravitational field. We have introduced the ES log-normal distribution fully determined by the Einstein median as the scale parameter and the Shannon shape parameter $\sigma = 1/\sqrt{6}$. Shannon formulated this shape parameter for the log-normal distribution describing systems with the maximum entropy formation. This ES log-normal distribution function determines the most effective mutual interactions between the gravitational field and the Maxwell-Boltzmann particles. In order to make the Einstein median formula more general, we have introduced the model of the active solid angle of the source of gravity with values $1 \leq \Omega \leq 4$ steradians. We have tested this ES log-normal distribution with three datasets measured on the Solar disc and two datasets measured on the surface of the Earth using the Mössbauer effect. There were predicted some new properties of those datasets. This model might stimulate and promote new initiatives to collect new better datasets for the Solar disc and the Mössbauer effect.

Keywords — ES log-normal distribution, Einstein median, Mössbauer effect data, Shannon shape parameter, Solar gravitational redshift data.

I. INTRODUCTION

The Solar gravitational redshift ($\text{GRS}_{\odot}$) was discovered by Lewis E. Jewell in 1896 [1], [2]. The Solar limb effect was discovered by J. Halm in 1907 [3]. The very intensive experimental and theoretical research was given to this topic by many scholars, the leading interpretation is based on the general theory of relativity of Albert Einstein predicting the solar gravitational redshift as $z = 2.11 \times 10^{-6}$ and the redshifted speed at the Sun as $v_{\text{GRS}_{\odot}} = 636.31 \text{ m s}^{-1}$ and the redshifted speed observed at the Earth $v_{\text{GRS,theo}} = 633.10 \text{ m s}^{-1}$ [4]. There are known many attempts to interpret the observed spread of Solar gravitational redshifts [5]–[19].

We have discovered the key for our model in the quote of George F. FitzGerald who reacted to the discovery of the Solar gravitational redshift in 1897 as [20]: “Everybody must feel the very greatest interest in this work. It is bringing us measurably nearer a knowledge of atomic movements and interactions, the great goal of modern physical research.”

We have extracted from the FitzGerald’s quote following: the Solar gravitational field interplays with all Maxwell-Boltzmann particles under some hidden universal rules that can be measured by the redshifted Fraunhofer lines. Can we discover this hidden universal distribution function?

II. BRIEF REVIEW OF OBSERVATIONS ON THE SOLAR DISC

There were published many quantitative observations of the Solar gravitational redshifts. The dominant approach of almost all observations was to compare the observed gravitational redshifts with the prediction of Albert Einstein. Table I summarizes some published data on the redshifted speed written in the redshifts of Fraunhofer lines.

The final determination of the value of the Solar gravitational redshift depends on the selection of observed Fraunhofer lines and their position on the Solar disc. We will study the properties of our model on some historical datasets with a large number of measured lines without any selection. In our model, all

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Fraunhofer lines contribute to the final gravitational interaction with the Solar gravitational field. The final gravitational attraction is determined by the redshifted speeds of all Maxwell-Boltzmann particles.

TABLE I: THE DETERMINATION OF THE SOLAR GRAVITATION REDSHIFT IN THE LAST 60 YEARS

Year	$R \equiv \Delta_{\text{obs}}/\Delta_{\text{theory}}$	Number of lines	Selection of lines	Reference
1961	$R \approx 0.5$ (center)	Line Sr I 4607	yes	[21]
1970	$R = 0.61 \pm 0.06$	Line K I 7699	yes	[22]
1972	$R = 1.01 \pm 0.06$	Line K I 7699	yes	[23]
1977	$R = 0.97 \pm 0.02$	Line Ti I 5713	yes	[24]
1980	$R = 0.76 \pm 0.24$	738 lines Fe I	no	[25]
1980	$R = 0.97 \pm 0.16$	738 lines Fe I	Only 71 lines	[25]
1991	$R = 0.73$	Oxygen triplet	yes	[26]
1998	Depends on the selection	1446 Fe I lines	Dataset of 1446 Fe I lines	[27]
2012	Histogram of redshifted speeds	2334 lines	9 selected lines gives the theoretical value	[28]
2012	$R = 1.10 \pm 0.19$	5188-5212 Å	yes	[29]
2014	$R = 0.95$	Line K I 7699	Observed 1976-2013	[30]
2020	$R = 1.009 \pm 0.009$	326 Fe I lines	Selected 97 lines	[4]

III. SIX HISTOGRAMS OF THE SOLAR GRAVITATIONAL REDSHIFTS

We can get a good overview about the state of the situation in this field through the study of historical histograms. Fig. 1 depicts the Solar gravitational redshifted speeds based on the experimental data of Lewis E. Jewell who discovered the Solar gravitational redshift in 1896 [1], [2].

Fig. 1. Discovery of the Solar gravitational redshift by Jewell in 1896, 125 lines of different chemical elements recalculated as the redshifted speeds with the average value $v_{\text{GRS,theo}} = 686 \text{ m s}^{-1}$.

Fig. 2 shows the histogram of Leonhard Grebe and Albert Bachem who cooperated with Albert Einstein around the year 1920 [31]-[33].

Fig. 2. Grebe – Bachem histogram in 1920, the selection of 25 lines from 124 lines with the average value $v_{\text{GRS,theo}} = 540 \text{ m s}^{-1}$.

Fig. 3 brings an example of the histogram of Gonzáles Hernández from 2020 [4] where 97 Fe I lines were selected from 326 lines. These measured data were taken from light reflected from the surface of the Moon.

In the historical literature, we have studied two histograms of Charles Edward St. John [34] who was a top solar spectroscopist of that epoch working at the Mount Wilson Solar Observatory – Fig. 4 and Fig. 5 based on the data from 1928.

Fig. 5 brings the Solar gravitational redshift taken by St. John [34] close to the edge of the Solar disc to measure the Solar limb effect.

Fig. 3. Histogram of Gonzáles Hernández [4] in 2020, 97 Fe I lines were selected from 326 lines with the average redshifted speed $V_{GRS,theo} = 639 \pm 14 \text{ m s}^{-1}$.

Fig. 4. St. John histogram of 586 Fe I lines [34] observed in the center of the Solar disc, the MODE redshifted speed $V_{GRS,theo} = 500 \text{ m s}^{-1}$.

Fig. 5. St. John in 1928 [34] observed 133 Fe I lines close to the edge of the Solar disc at $0.985 R_{\odot}$, the MODE redshifted speed $V_{GRS,theo} = 800 \text{ m s}^{-1}$.

Walter S. Adams in 1910 [35] was inspired by the Solar limb effect discovered by J. Halm in 1907 [3]. Adams was also the top solar spectroscopist of that epoch working at the Mount Wilson Solar Observatory and very well equipped with instruments. Adams extracted his data closer to the edge of the Solar disc in comparison with St. John. Therefore, this dataset is more redshifted compared to two histograms of St. John.

Fig. 6. Adams in 1910 observed 470 lines of different chemical elements on the edge of the Solar disc at $0.998 R_{\odot}$, the MODE redshifted speed $v_{GRS,theo} = 1040 \text{ m s}^{-1}$.

IV. EINSTEIN MEDIAN AND THE ACTIVE SOLID ANGLE Ω OF THE SUN

Albert Einstein derived in 1915 the formula for the Solar gravitational redshift as:

$$\frac{\lambda}{\lambda_0} = \frac{1}{\sqrt{1 - \frac{2GM_{\odot}}{R_{\odot}c^2}}} \approx 1 + \frac{GM_{\odot}}{R_{\odot}c^2} \quad (1)$$

In order to make the formula for the Einstein median more general, we propose to insert into the Einstein formula the active solid angle Ω that appears under some gravitational situations with the value different from $\Omega = 2$ steradians. This more general formula can be found in the paper of Emil Wiechert in 1920 [36]:

$$\frac{\lambda}{\lambda_0} = \frac{1}{\sqrt{1 - \frac{\Omega GM_{\odot}}{R_{\odot}c^2}}} \approx 1 + \frac{\Omega}{2} \frac{GM_{\odot}}{R_{\odot}c^2} \quad \Omega = 1, 2, 3, 4 \quad (2)$$

The median redshifted speed $v_{GRS \text{ MEDIAN}}$ for the set of the Maxwell-Boltzmann particles is given as:

$$v_{GRS \text{ MEDIA}} = \frac{\Delta\lambda}{\lambda_0} c = \frac{\Omega}{2} \frac{GM_{\odot}}{R_{\odot}c} \quad (3)$$

The individual Maxwell-Boltzmann particles might be redshifted as:

$$v_{GRS \text{ theo}} = \omega \frac{\Delta\lambda}{\lambda_0} c = \omega \frac{\Omega}{2} \frac{GM_{\odot}}{R_{\odot}c} \quad 0 \leq \omega \quad (4)$$

For the experimental test of this model, we have to collect redshifted speeds for all Maxwell-Boltzmann particles without any selection. The dataset should be taken on a well-defined position on the Solar disc because there might be measured a “blend” of redshifted data from different regions. The dataset of observed redshifted speeds should be as large as possible.

The active solid angle Ω can be studied in the gravitational situations given in Table II.

TABLE II: THE ACTIVE SOLAR ANGLE Ω IN GRAVITATION SITUATIONS

The active solid angle Ω	The gravitational experiment
$\Omega = 1$	Transverse Mössbauer effect
$\Omega = 2$	Longitudinal Mössbauer effect
$\Omega = 2$	Fraunhofer lines in the center of the Solar disc ($v_{\text{GRS, MEDIAN}} = 633.10 \text{ m s}^{-1}$)
$\Omega = 3$	Fraunhofer lines near to the edge of the Solar disc ($v_{\text{GRS, MEDIAN}} = 949.65 \text{ m s}^{-1}$)
$\Omega = 4$	Fraunhofer lines on the edge of the Solar disc ($v_{\text{GRS, MEDIAN}} = 1266.20 \text{ m s}^{-1}$)

V. THE ES LOG-NORMAL DISTRIBUTION

The Solar gravitational field is acting on all Maxwell-Boltzmann particles and causing the observed spread of redshifted speeds of individual particles towards the center of the Sun. Microscopically there should be observed a universal distribution function of those redshifted speeds but macroscopically the test mass, composed from a huge number of Maxwell-Boltzmann particles, obeys a scale parameter directing the move towards the source of the gravitational field.

For the description of the log-normal distribution we have to discover two universal parameters: the first determines the position of the log-normal distribution – the scale parameter = the EINSTEIN MEDIAN. The second parameter determines the shape of the log-normal distribution curve. Claude Shannon derived for the log-normal distribution the SHANNON SHAPE PARAMETER $\sigma = 1/\sqrt{6} \approx 0.4082$ based on the extremal principle of entropy. There are a lot of situations where this Shannon shape parameter determines very well the observed experimental data, e.g., [37] where is a very good introduction to this Shannon model. We assume that the Solar gravitational field interplays with all Maxwell-Boltzmann particles and the combination of microscopic redshifted speeds of individual particles results in a single macroscopic redshift speed determined by the Einstein median. The Shannon shape parameter determines the most effective cooperation between the Solar gravitational field and the individual Maxwell-Boltzmann particles of an object near the Sun. There is one more parameter “hidden” in the Einstein median – the value of the “hidden” third parameter - the ACTIVE SOLID ANGLE PARAMETER of the gravitational source – Ω should be inserted in the ES log-normal distribution in steradians. Based on the Einstein prediction scholars use only the value $\Omega = 2 \text{ sr}$ but there are some gravitational situations where the active solid angle is $\Omega = 1, 2, 3, 4 \text{ sr}$.

Table III summarizes some properties of the ES log-normal distribution.

TABLE III: THE PROPERTIES OF THE ES LOG-NORMAL DISTRIBUTION

NOTATION	LOG-NORMAL PROPERTIES
Solid angle parameter Ω	$= 1, 2, 3, 4$
Einstein scale parameter	$\mu = \ln \left(\frac{\Omega}{2} \frac{GM_{\odot}}{R_{\odot}c} \right)$
Shannon shape parameter	$\sigma = \frac{1}{\sqrt{6}} \approx 0.4082$
PDF	$\frac{1}{x\sigma\sqrt{2\pi}} \exp \left(-\frac{(\ln(x) - \mu)^2}{2\sigma^2} \right)$
PROPERTIES FOR $\Omega = 2$	
MODE	$\exp(\mu - \sigma^2) \approx 535.9$
MEDIAN	$\exp(\mu) \approx 631.1$
MEAN	$\exp \left(\mu + \frac{\sigma^2}{2} \right) \approx 688.1$
SKEWNESS	$(\exp(\sigma^2) + 2) \sqrt{\exp(\sigma^2) - 1} \approx 1.355$
KURTOSIS	$\exp(4\sigma^2) + 2\exp(3\sigma^2) + 3\exp(2\sigma^2) - 6 \approx 3.432$
ENTROPY	$\mu + \frac{1}{2} + \ln(\sigma\sqrt{2\pi}) \approx 6.974nat$
INFLECTION POINTS	$\exp \left(\mu - \frac{3}{2}\sigma^2 \pm \frac{1}{2}\sigma\sqrt{\sigma^2 + 4} \right) \approx 325.0 \text{ and } 747.9$
GEOMETRIC STANDARD DEVIATION	$\exp(\sigma) \approx 1.504$
GEOMETRIC VARIANCE	$\exp(\sigma^2) \approx 1.181$
GEOMETRIC COEFFICIENT OF VARIATION	$\exp(\sigma) - 1 \approx 0.504$
ARITHMETIC STANDARD DEVIATION	$\exp \left(\mu + \frac{1}{2}\sigma^2 \right) \sqrt{\exp(\sigma^2) - 1} \approx 293.0$
COEFFICIENT OF VARIATION	$\sqrt{\exp(\sigma^2) - 1} \approx 0.4258$

The ES log-normal distribution determined by the Einstein median redshifted speed $v_{\text{GRS MEDIAN}} = 631.1 \text{ m s}^{-1}$ (the scale parameter), the Shannon shape parameter $\sigma = 1/\sqrt{6}$, and the “hidden” parameter $\Omega = 2 \text{ sr}$ is given on Fig. 7.

Fig. 7. The ES log-normal distribution with parameters describing the redshifted speed of the Maxwell-Boltzmann particles in the center of the Solar disc.

Under some experimental situations we cannot get experimental data for all parts of the log-normal distribution. At these situations it is possible to get the complete ES log-normal distribution from the determined values of inflection points L1 and L2 – Fig. 8.

Fig. 8. The inflection points L1 and L2 could recover the complete ES log-normal distribution from an incomplete histogram.

VI. THE ES LOG-NORMAL DISTRIBUTION IN REAL SITUATIONS

We have tested the ES log-normal distribution on three datasets taken by top solar spectroscopists at the Mount Wilson Solar Observatory equipped with the best technology of their epoch – Walters S. Adams collected a large dataset of redshifted Fraunhofer lines on the edge of the Solar disc in 1910 and Charles Edward St. John published in 1928 two large datasets of redshifted of Fe I lines in the center of the Solar disc and close to the edge of the Solar disc.

We have evaluated from their histograms the MODE redshifted speed and recalculated as the MEDIAN redshifted speed, the shape parameter of their distribution we have determined from LN distribution of the individual redshifted speeds. The “hidden” parameter Ω was selected for that measured position on the Solar disc in order to match the observed data better than $\Omega = 2$.

Fig. 9. ES log-normal distribution based on the dataset of St. John in 1928 [34]: 586 Fe I lines measured in the center of the Solar disc, MODE redshifted speed 500 m s⁻¹, MEDIAN redshifted speed 590 m s⁻¹, the shape parameter $\sigma = 0.5119$, the Solar active angle $\Omega = 2$.

Fig. 10. ES log-normal distribution based on the dataset of St. John in 1928 [34]: 133 Fe I lines close to the edge of the Solar discs at 0.985 R, MODE redshifted speed 800 m s⁻¹, MEDIAN redshifted speed 945 m s⁻¹, the shape parameter $\sigma = 0.2833$, the Solar active angle $\Omega = 3$, (very small dataset).

Fig. 11. ES log-normal distribution based on the dataset of Adams in 1910 [35]: 470 lines of different chemical elements on the edge of the Solar discs at 0.998 R, MODE redshifted speed 1040 m s⁻¹, MEDIAN redshifted speed 1228 m s⁻¹, the shape parameter $\sigma = 0.4764$, the Solar active angle $\Omega = 4$.

In order to better evaluate this model, it will be necessary to extract from the existing datasets larger sets of the redshifted Fraunhofer lines but with a very well-defined position on the Solar disc in order to avoid the “mixture” of redshifted Fraunhofer lines formed with different values of the active solid angle Ω . On the other side, it will be necessary to add to these distribution analyses Fraunhofer lines from ultraviolet and infrared regions of spectra.

VII. THE ES LOG-NORMAL DISTRIBUTION AND THE MÖSSBAUER EFFECT

The measurement of the gravitational redshift using the Mössbauer effect brought a new impulse to this field [15]. We can express the observed gravitational redshift on the dependence of the height h between the source and the absorber as:

$$z = \frac{\Delta\lambda}{\lambda_1} = \frac{gh}{c^2} = h \cdot 1.0896 \cdot 10^{-16} \quad (5)$$

or as the gravitationally redshifted speed $v_{GRS \text{ MEDIAN}}$ in $\text{nm s}^{-1} \text{ m}^{-1}$:

$$v_{GRS \text{ MEDIA}} = zc = \frac{gh}{c} = h \cdot 32.69 \left[\text{nm s}^{-1} \text{ m}^{-1} \right] \quad (6)$$

Two teams of researchers significantly contributed to this topic – one team is very famous, however, the second one is almost forgotten. We can rediscover a very important information in the forgotten paper of T.E. Cranshaw and J.P. Schiffer [38]. They placed the source to the top position and the absorber to the bottom and found the value 0.859 ± 0.085 of the Einstein prediction. We can express this experimental value as:

$$v_{GRS \text{ MODE}} = v_{GRS \text{ MEDIAN}} \cdot \exp(\sigma^{-2}) = 0.8465 v_{GRS \text{ MEDIAN}} \quad (7)$$

This interesting asymmetry in the data might reveal some deeper knowledge about the gravitational redshifts. This situation is depicted in Fig. 12.

Fig. 12. Cranshaw – Schiffer observation of the gravitational redshift with the source above the absorber [38]

Therefore, we studied in details the works of Glen A. Rebka, Robert V. Pound and Joseph L. Snider in order to find similar asymmetry in their data. We have found such asymmetry in their Table I in 1960 in data before the temperature corrections [39] and in their paper [40] on page B 802. The gravitational redshift depends on the position of the source and the absorber – Fig. 13.

Fig. 13. Pound, Rebka, Snider data [39], [40] based on the position of the source and the absorber.

We can write a formula for two-way inspection of the gravitational redshift:

$$v_{GRS \text{ TWO-WAY}} = \sqrt{v_{GRS \text{ MEDIAN}} \cdot \exp(\sigma^{-2}) v_{GRS \text{ MEDIAN}} \cdot \exp(\sigma^2)} = v_{GRS \text{ MEDIAN}} \quad (8)$$

This is the reason why we will get in two-way observations of the gravitational redshift of the Einstein median with very high accuracy, e.g. [41]-[46].

VIII. THE NEWTONIAN GRAVITATIONAL CONSTANT AS THE MACROSCOPIC RESULT OF MICROSCOPIC EVENTS

There is one interesting conclusion of this contribution: we might newly view the Newtonian gravitational constant [47]-[59] as the macroscopic picture of microscopic events. For the active solid angle $\Omega = 2$, we can use the Einstein median as a representative quantity for all redshifted speeds in the gravitational field of the Sun:

$$v_{GRS \text{ MEDIAN} \odot} = \frac{\Delta \lambda}{\lambda_0} c = \frac{GM_{\odot}}{R_{\odot} c} \approx 636.31 \text{ ms}^{-1} \quad (9)$$

$$G = \frac{v_{GRS \text{ MEDIAN} \odot} \cdot c \cdot R_{\odot}}{M_{\odot}} = G \quad (10)$$

For the Earth with the redshifted speed $v_{GRS \text{ MEDIAN} \oplus}$ on the surface we can write the formula as:

$$v_{GRS \text{ MEDIAN} \oplus} = \frac{\Delta \lambda}{\lambda_0} c = \frac{GM_{\oplus}}{R_{\oplus} c} \approx 0.2085 \text{ ms}^{-1} \quad (11)$$

The acceleration g on the surface of the Earth may be written as:

$$g = v_{GRS \text{ MEDIAN} \oplus} \frac{c}{R_{\oplus}} = \frac{GM_{\oplus}}{R_{\oplus}^2} \approx 9.8 \text{ ms}^{-2} \quad (12)$$

The Newtonian gravitational constant G for the Earth might be written as:

$$G = \frac{v_{GRS \text{ MEDIAN} \oplus} \cdot c \cdot R_{\oplus}}{M_{\oplus}} = G \quad (13)$$

Finally, we get this expression for the Newtonian Formula for the force as:

$$F = G \frac{M_{\oplus} m}{R_{\oplus}^2} = \frac{v_{GRS \text{ MEDIAN } \oplus} \cdot c}{R_{\oplus}} m = gm \quad (14)$$

Equation 14 unites three Great Masters: Galileo, Newton and Einstein. Equation 14 describes the same gravitational event seen both from the Macrocosmos and Microcosmos.

IX. CONCLUSION

We might open a new road leading towards the model of quantum gravity. This contribution could stimulate some new activities of the international spectroscopic community to reanalyze all existing data on the gravitational redshifts and to collect more precise and better-defined data with our existing technology. There is space enough for all participants on this Project.

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CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

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