How to Decipher the Seegers-Tisserand-Gerber-Einstein Formula and the Soldner-Einstein Formula?

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Abstract — The famous Seegers-Tisserand-Gerber-Einstein Formula describing correctly the Mercury perihelion advance passed through hands of many scholars who tried to decipher the physical meaning of the perturbation factor Ω introduced by Carl Seegers in 1864. Based on the Gauss's law for gravity we have newly interpreted this perturbation factor Ω as the active solid angle of the Solar gravitational field $\Omega = 3$ steradians. We have inserted this model of the active solid angle of the Solar gravitational field the famous Soldner-Einstein Formula describing the light deflection in the vicinity of the Sun with $1 \leq \Omega \leq 8$. The enormous scatter of experimental data of the light deflections measured during the Solar eclipses was interpreted as the quantum jumps of the deflection angle with the quantum jump $0''.44$. All known existing data on the light deflection taken during the last hundred years were depicted into the graphs. In some cases we have discovered these quantum jumps of the deflection angle during the individual runs of the Solar eclipse experiment. We propose to reanalyze all historical data taken for individual stars and to search for a hidden structure in these data. Moreover, we want to initiate new experimental activities for the coming Solar eclipses in order to collect more precise data that might guide us towards the model of quantum gravity.

Keywords — Active solid angle, light deflection, Mercury perihelion advance, quantum gravity, quantum jumps.

I. INTRODUCTION

The solution of difficult physical problems develops in three stages: Stage I: Both the mathematical formula and the physical model are wrong. Stage II: The mathematical formula is correct but the physical interpretation is wrong. Stage III: Both the mathematical formula and the physical model are correct.

The light deflection in the vicinity of the Sun (e.g., [1]-[4]) and the Mercury perihelion advance (e.g., [5]-[6]) played a very important role in the development of physics. Are we now with these topics in the Stage II or the Stage III? Can we add anything reasonable to these deeply studied Formulae?

II. EVOLUTION OF THE SEEGER-TISSERAND-GERBER-EINSTEIN FORMULA (STGE FORMULA)

The first important step in the evolution of the STGE Formula made Carl Seegers in his PhD Thesis under the guidance of Wilhelm Weber in 1864 [7]. Carl Seegers stated: "Using Weber's law, the orbit of the body deviates from the elliptical one and the perihelion of the attracted body moves in a circle around the Sun. After one orbit the perihelion shifts by this value" (written in the modern notation):

$$\omega = 2\pi \Omega \frac{GM_{\odot}}{c^2 a (1 - e^2)} \quad (1)$$

where ω is the perihelion advance after one orbit, Ω is an un-known perturbation factor, G is the Newtonian gravitational constant, $M_{\odot}$ is the Solar mass, c is the speed of gravity (Weber used the value $\sqrt{2} c$ in his model), a is the semi-major axis, and e is the eccentricity of that ellipse.

This was the starting formula for the interpretation of the famous experimental value $43''$ found by Le Verrier. The main problem was to interpret that un-known perturbation factor Ω , and many solutions were proposed [5]. There were developed several concepts how to find a physical interpretation for Ω – Table I.

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TABLE I: THE EVOLUTION OF THE SEEGER-TISSERAND-GERBER-EINSTEIN FORMULA

Year	Scholar	Perturbation coefficient Ω	Physical interpretation	Predicted advance	Reference
1864	Seegers	1	Weber's law with $\sqrt{2} c$	7".17	[7]
1872	Tisserand	1	Weber's law with c	14".3	[8]
1890	Tisserand	1	Gauss's law with c	14".3	[9]
1898	Gerber	3	Weber's law with retarded potentials and with c	43"	[10]
1915	Einstein	3	General relativity	43"	[11]
2021	This paper	3	Gauss's law for gravity with the active solid angle $\Omega = 3$ sr	43"	

In our model the integral form of Gauss's law for gravity states:

$$\frac{\Omega}{4\pi} \oint \mathbf{g} \cdot d\mathbf{A} = -\Omega GM \quad (2)$$

where Ω in steradians represents the active solid angle of the gravitational source for that gravitational situation. Other symbols are taken from the well known Gauss's law for gravity: a surface integral over a closed surface, $\mathbf{g}$ is the gravitational field, $d\mathbf{A}$ is a vector, whose magnitude is the area of an infinitesimal piece of surface, G is the Newtonian gravitational constant, M is the total mass enclosed within the surface.

Fig. 1 shows the gravitational situation for the $\Omega = 1$ steradian.

Fig. 1. The gravitational field with the active solid angle $\Omega = 1$ steradian (from Wikimedia with the permission licence: https://upload.wikimedia.org/wikipedia/commons/9/9b/Steradian_V2.svg).

There are different gravitational situations in which the source of the gravitational field acts on the attracted body. For the case of the Mercury perihelion advance we propose to insert into the Seegers-Tisserand-Gerber-Einstein Formula the active solid angle $\Omega = 3$ sr. This situation is depicted in Fig. 2.

Fig. 2. The Solar gravitational field acting via $\Omega = 3$ steradians on the Mercury perihelion advance, L is the semi-latus rectum of that ellipse, a is the semi-major axis, e is the eccentricity.

III. LIGHT DEFLECTION IN THE VICINITY OF THE SUN

There are three different gravitational experiments for collecting experimental data of the photon deflection under the influence of the Solar gravitational field. We can sort them in the dependence of the intensity of the Solar gravitational field and the mass of deflected photons. Table II summarizes these three gravitational observations.

TABLE II: THREE GRAVITATIONAL EXPERIMENTS IN THE SOLAR SYSTEM ON THE LIGHT BENDING

Intensity of the Solar gravitational field	Photon mass	Method	Active solid angle Ω [sr]	Reference
HIGH	LOW ($\lambda \approx 5 \cdot 10^{-2}$ m)	VLBI	4	[12]
LOW	HIGH ($\lambda \approx 5 \cdot 10^{-7}$ m)	HIPPARCOS	4	[12]
HIGH	HIGH ($\lambda \approx 5 \cdot 10^{-7}$ m)	Solar eclipse	$1 \leq \Omega \leq 8$	

A deflection of light rays must take place if the gravitation influences the speed of light. This model was introduced by Albert Einstein in his Prague paper in 1911 [13] and later in the Schwarzschild metrics as:

$$c_R = c_\infty \left(1 - \frac{2GM}{c^2 R} \right) \quad (3)$$

where the light speed c_R depends on the distance R from the centre of the gravitational source and c_∞ is the far away distance..

Emil Wiechert in 1920 proposed to make this formula more general [14]:

$$c_R = c_\infty \left(1 - \Omega \frac{GM}{c^2 R} \right) \quad (4)$$

From this (4) we will get a more general expression for the light bending in the vicinity of the Sun:

$$\theta = 1 - \frac{c_R}{c_\infty} = \Omega \frac{GM_\odot}{c^2 R_\odot} \quad 1 \leq \Omega \leq 8 \quad (5)$$

We predict the existence of quantum jumps with the deflection angle jump $0''.44$. There was already published one attempt by G.G. Nyambuya [15] to quantize the deflection angle jumps with the quantum jump $0''.87$. G.G. Nyambuya's interpretation was based the influence of the photon spin on these quantum jumps.

This gravitational effect can be observed during the Solar eclipse events where both the Solar gravitational field and the photon mass are high enough to stimulate these quantum jumps.

Fig. 3. Deflection of the light in the vicinity of the Sun with $\Omega = 4$ steradians (Einstein shift).

IV. ALL DATA FROM THE SOLAR ECLIPSE EXPEDITIONS

Many teams of professional astronomers took valuable data in the time span 1919 -1973. However, the scatter of data was so high that no reasonable structure in these data was discovered and these professional activities were stopped in 1973. Table III surveys all these data for the light bending observed during the Solar eclipses.

TABLE III: DATA OF THE SOLAR ECLIPSE EXPEDITIONS

Number	Date	Location	Average deflection [arcsecond]	Reference
1	May 29, 1919	Sobral	1.98 ± 0.16	[16]
2	May 29, 1919	Principe	1.61 ± 0.40	[16]
3	September 21, 1922	Wallal	1.72 ± 0.11	[17]
4	September 21, 1922	Wallal	1.82 ± 0.15	[17]
5	September 21, 1922	Wallal	1.74 ± 0.3	[18]
6	September 21, 1922	Condillo	1.77 ± 0.3	[19]
7	May 9, 1929	Takengon	2.24 ± 0.10	[20]
8	June 19, 1936	Kubishev	2.73 ± 0.31	[21]
9	June 19, 1936	Kosimizu	1.71 ± 0.40	[22]
10	May 20, 1947	Brazil	2.01 ± 0.27	[23]
11	February 25, 1952	Sudan	1.70 ± 0.10	[24]
12	October 02, 1959	Sahara	2.17 ± 0.34	[25]
13	February 15, 1961	Italy	1.98 ± 0.46	[26]
14	June 30, 1973	Mauritania	1.66 ± 0.18	[27]
15	August 21, 2017	USA	1.751 ± 0.060	[28]

In 2020 Goldoni and Stefanini published the database for the light deflections of 170 individual stars taken during many of these expeditions with $\theta_{\text{average}} = 1''.98$ [29]-[30].

$$\text{this model } \theta_{\text{average}} = 0.44 + \frac{8-1}{2} \times 0.44 = 1''.98$$

The dataset, published by Emanuele Goldoni [30] for 170 individual stars, was very inspirational for our research, and we tried to discover a hidden structure in that chaotic scatter of data. We have depicted individual stars for a given Solar eclipse and observed their quantum jumps of deflection angles in Fig. 4–14.

Fig. 4. Solar eclipse on May 29, 1919. Data from reference [30].

Fig. 5. Solar eclipse on September 21, 1922. Data from reference [30].

Fig. 6. Solar eclipse on May 9, 1929. Data from reference [30].

Fig. 7. Solar eclipse on June 19, 1936. Data from reference [30].

Fig. 8. Solar eclipse on June 19, 1936. Data from reference [21].

Fig. 9. Solar eclipse on May 20, 1947. Data from reference [30].

Fig. 10. Solar eclipse on February 25, 1952. Data from reference [30].

Fig. 11. Solar eclipse on October 02, 1959. Data from reference [25].

Fig. 12. Solar eclipse on February 15, 1961. Data from reference [26].

Fig. 13. Solar eclipse on June 30, 1973. Data from reference [30].

Fig. 14. Solar eclipse on August 21, 2017. Data from reference [30].

V. QUANTUM JUMPS DURING THE RUNNING TIME OF THE SOLAR ECLIPSE

The real situation with the interpretation of the light bending is more complicated. During the detailed analysis of several plates taken during the single Solar eclipse we can see in data “quantum jumps” of the deflection angles. Freundlich, von Klüber and von Brunn [20] made the very successful measurement on May 9, 1929 and took four plates during the single Solar eclipse. Their data are given in Table IV.

TABLE IV: THE QUANTUM JUMPS OBSERVED IN THE RUNNING SOLAR ECLIPSE EXPERIMENT [20]

Time since START	Plate	Deflection angle [arcsecond]	Ω [sr]
10 seconds waiting			
40 seconds exposure	FT ₄₀	2.25 ± 0.19	5
15 seconds change			
90 seconds exposure	FT ₉₀	2.17 ± 0.20	5
15 seconds change			
60 seconds exposure	FT ₆₀	2.61 ± 0.26	6
15 seconds change			
40 seconds exposure	FT _{40a}	1.81 ± 0.19	4
10 seconds waiting			
END			

Interpretation: quantum jumps: $5 \rightarrow 5 \rightarrow 6 \rightarrow 4$

The detailed inspection of all data revealed a possible hidden structure in those light bending angles. For a better understanding of this structure we have to get new more precise data with the existing technology in our epoch.

VI. PREDICTIONS FOR THE LIGHT BENDING IN THE VICINITY OF THE SUN

Since 1801 there were done several predictions for the value of the light bending close to the surface of the Sun. These predictions are given in Table V.

TABLE V: PREDICTIONS FOR THE LIGHT BENDING IN THE VICINITY OF THE SUN

Year	Scholar	Interpretation	Predicted value of deflection angle	Active solid angle Ω [sr]	Reference
1801	Soldner	Newtonian gravity	$0''.87$	2	[1]
(1909)	Ritz	Ballistic theory	$1''.31$	3	[32]
1915	Einstein	Einsteinian gravity	$1''.75$	4	[33]
(1898)	Gerber	Webers's law with retarded potentials	$2''.62$	6	[34]
2021	This paper	Gauss's law for gravity with active solid angle	$0''.44 \leq \theta \leq 3''.52$	$1 \leq \Omega \leq 8$	

VII. CLASSROOM ACTIVITY OF GOLDONI AND STEFANINI

Goldoni and Stefanini [29] originally created their database as a possible activity for high-school students. It might happen that this activity could be very important for scholars searching for a model of

quantum gravity.

The target of this lesson is as follows [29]: “Two famous scientists A and B have proposed two alternative theories for an important astronomical phenomenon. According to A, the actual value should be 1”.75; on the contrary, B’s theory leads to 0“.87. The scientific community is split: different teams around the world have been asked to carry out experiments and gather useful data. Today we have received all the values obtained on the field. Now we have been asked to analyse them and choose which theory is the right one (or if both are wrong). The world is waiting for us!”

VIII. CONCLUSION

We might open a new road leading towards the model of quantum gravity. This contribution could stimulate some new activities of the international astronomical community to reanalyze all existing data on the bending of light in the vicinity of the Sun and to start preparation for the coming Solar eclipses to get more precise data with our existing technology. There is space enough for all participants on this Project.

However, K.J. Treschmann quoted [31]: “Anyone, no matter how well prepared can experience cloud at the crucial time of an eclipse.”

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CONFLICT OF INTEREST

Authors declare that they do not have any conflict of interest.

REFERENCES

- [1] Jaki SL. Johann Georg von Soldner and the gravitational bending of light, with an English translation of his essay on it published in 1801. *Foundations of Physics*. 1978;8:927-950.
- [2] Ginoux JM. Albert Einstein and the doubling of the deflection of light. *Foundations of Science*. 2021; <https://doi.org/10.1007/s10699-021-09783-4>.
- [3] Sauer T. Soldner, Einstein, gravitational light deflection and factors of two. *Annalen der Physik*. 2021; 533: 2100203. DOI: 10.1002/andp.202100203.
- [4] Lotze KH, Simonato S. Henry Cavendish and the effect of gravity on propagation of light: postscript. *Eur. Phys. J. H*. 2021; 46:24. <https://doi.org/10.1140/epjh/s13129-021-00027-4>.
- [5] Roseveare NT. *Mercury’s perihelion From Le Verrier to Einstein*. Oxford: Clarendon Press; 1982.
- [6] Janssen M, Renn M. *Einstein and the perihelion motion of Mercury*. 2021; Arxiv: <https://arxiv.org/abs/2111.11238>.
- [7] Seegers, C. De motu perturbationibusque planetarum secundum logem electrodynamicam Weberianam solem ambientum. Inaugural Diss. 1864; Göttingen. (On the motion and perturbations of the planets circling the Sun according to the electrodynamic law of Weber). *Über die Bewegung und die Störungen der Planeten, wenn dieselben sich nach dem Weberschen elektrodynamischen Gesetz um die Sonne bewegen*. Braunschweig: Vieweg & Sohn, 1924. Chapter 5.
- [8] Tisserand F. Sur le mouvement des planètes autour du soleil, d’après la loi électrodynamique de Weber. (On the motion of planets around the Sun according to Weber’s electrodynamic law). *Comptes Rendues de l’Academie des Sciences de Paris*. 1872; 75: 760-763.
- [9] Tisserand F. Sur les mouvements des planètes, en supposant l’attraction représentée par l’une des lois électrodynamique de Gauss ou de Weber. (On the motion of planets when one supposes that the attraction is represented by either electrodynamic law of Gauss or that of Weber). *Comptes Rendues de l’Academie des Sciences de Paris*. 1890; 110: 313-315. (See also discussion in Reference [5], page 126).
- [10] Gerber P. Die räumliche und zeitliche Ausbreitung der Gravitation. (The spatial and temporal propagation of gravity). *Zeitschrift für Mathematik und Physik II*. 1898; 43: 93-104.
- [11] Einstein A. Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie. (Explanation of the perihelion motion of Mercury from the general theory of relativity). *Königlich Preussische Akademie der Wissenschaften (Berlin). Sitzungsberichte*: 1915; 831-839.
- [12] Will, CM. The confrontation between general relativity and experiment. *Living Reviews in Relativity*. 2014; 17: Article number 4.
- [13] Einstein, E. Einfluss der Schwerkraft auf die Ausbreitung des Lichtes. (On the influence of gravitation on the propagation of light). *Annalen der Physik*. 1911; 4(35): 898-908.
- [14] Wiechert, E. Die Gravitation als elektrodynamische Erscheinung. (Gravity as an electrodynamic phenomenon). *Annalen der Physik*. 1920; 63: 301-381, page 317.
- [15] Nyambuya, G.G. A prediction of quantized gravitational deflection of starlight. *Perspectives Journal*. 2016; 7(13): 1827-1833.
- [16] Dyson FW, Eddington, A, Davidson, CR. A determination of the deflection of light by the Sun’s gravitational field, from observations made at the total eclipse of May 29, 1919. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*. 1920; 220: 291-333.
- [17] Cambell, WW, Trumpler RJ. Observations on the deflection of light in passing through the Sun’s gravitational field. *Lick Observatory Bulletin*. 1923; 11: 41-54.
- [18] Chant, CA, Young, RK. Evidence of the bending of the rays of light on passing the Sun, obtained by the Canadian expedition to observe the Australian eclipse. *Publications of the Dominion Astrophysical Observatory Victoria*. 1923; 2: 275-285.

- [19] Dodwell, GF, Davidson, CR. Determination of the deflection of light by the Sun's gravitational field from observations made at Cardillo Downs, South Australia, during the total eclipse of 1922 September 21. *Monthly Notices of the Royal Astronomical Society*. 1924; 84: 150-162.
- [20] Freundlich, E, von Klüber, H, von Brunn, A. Ergebnisse der Potsdamer Expedition zur Beobachtung der Sonnenfinsternis von 1929, Mai 9, in Takengon (Nordsumatra). (Results of the Potsdam's expedition for the observation of the Solar eclipse on 1929, May 9, in Takengon (North Sumatra)). *Zeitschrift für Astrophysik*. 1931; 3: 171-198.
- [21] Mikhailov, AA. The deflection of light by the gravitational field of the Sun (George Darwin Lecture). *Monthly Notices of the Royal Astronomical Society*. 1959; 119: 593-608.
- [22] Matukuma, T. On the Einstein effect derived from the observations of the total Solar eclipse of June 19th in 1936. *Japanese Journal of Astronomy and Geophysics*. 1951; 18: 51-72.
- [23] Van Biesbroeck, G. The Einstein shift at the eclipse of May 20, 1947, in Brazil. *The Astronomical Journal*. 1950; 55: 49-53.
- [24] Van Biesbroeck, G. The relativity shift at the 1952 February 25 eclipse of the Sun. *The Astronomical Journal*. 1953; 58: 87-88.
- [25] Schmeidler, F. Neuer Versuch einer Messung der relativistischen Lichtablenkung. (New attempt of a measurement of relativistic light deflection). *Astronomische Nachrichten*. 1963; 287 (1): 7-16.
- [26] Schmeidler, F. Messung der Lichtablenkung während der Sonnenfinsternis am 15. Februar 1961. (Measurement of relativistic light deflection on February 15, 1961). *Astronomische Nachrichten*. 1985; 306 (2): 71-76.
- [27] Jones, BF. Gravitational deflection of light: Solar eclipse of 30 June 1973. – 2 plate reductions. *The Astronomical Journal*. 1976; 81: 455-463.
- [28] Bruns, DG. Gravitational starlight deflection measurements during the 21 August 2017 total Solar eclipse. *Classical and Quantum Gravity*. 2018; 35: 075009.
- [29] Goldoni, E, Stefanini, L. A century of light-bending measurements: bringing Solar eclipse into the classroom. *Physics Education*. 2020; 55(4): 045009.
- [30] Goldoni, E. Dataset of eclipses' measurements. <https://github.com/emanueleleg/eclipses>.
- [31] Treschmann, KJ. Early astronomical tests of general relativity: the gravitational deflection of light. *Asian Journal of Physics*. 2014; 23(1&2): 145-170.
- [32] Roseveare NT. *Mercury's perihelion From Le Verrier to Einstein*. Oxford: Clarendon Press; 1982. page 136.
- [33] Einstein, A. Die Feldgleichungen der Gravitation. (The field equations of gravity). *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*. 1915; 844-847.
- [34] Roseveare NT. *Mercury's perihelion From Le Verrier to Einstein*. Oxford: Clarendon Press; 1982. page 143.