Rotational Ground-State Bands of ^{168}Hf Nucleus

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ABSTRACT

This paper has been explained to study the rotational arrangement of even-even ^{168}Hf isotope using the phenomenological fitting, Sood's semi-empirical formula. The rotational energies from the calculated values were compared to the experimental data. The result shows that in ^{168}Hf , calculated energies fit the experimental values to a remarkable degree of accuracy.

Keywords: Rotational energy, semi-empirical formula, ^{168}Hf , Yrast state.

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I. INTRODUCTION

In the collective model proposed by Bohr and Mottelson [1], a new parameter called nuclear deformation is introduced, in which the surface of the nucleus may undergo oscillations in a rotating nucleus. These results in the prediction of rotational energy levels being restricted to even values due to oscillating symmetry and it is easily observed with nuclei having a number of nucleons far from closed shells. Experimental data comes to a good agreement with these predictions, and applications such as probability for B(E2) transitions, magnetic moments, quadrupole moments and isomeric transitions are so far successful.

In the macroscopic interpretation, the common simplest preliminary point of modeling the atomic nucleus is based on liquid drop model [2], [3] where the nucleus is assumed to behave a drop of dense incompressible liquid where the spherical shape is the result of attractive forces between all the particles contained in the nucleus. This model is then prolonged to explain quantum mechanical collective motion, for example vibrations and rotation, which is developed to the

Bohr and Mottelson collective model [1]. In these models, a new parameter called nuclear deformation is introduced, in which the surface of the nucleus may undergo oscillations in a rotating nucleus [4]. These results in the prediction of rotational energy levels being restricted to even values due to oscillating symmetry, and it is easily observed with nuclei having a number of nucleons far from closed shells.

There are lots of theoretical and experimental methods for rotational nuclei found in the literature. Jett *et al.* [5] were observed rotational band of Dysprosium (Dy), Erbium (Er), Ytterbium (Yb) and Hafnium (Hf) nuclei with neutron number 92, 96, 98, 102, respectively by excitation with neutron vaporization techniques. Panel *et al.* [6] studied rotational bands of low-level quasi-particle photon coupling plus rotor model of the ^{169}Er nucleus. Shimada *et al.* [7] have been studied rotational motion of triaxially deformed nuclei using the microscope angular momentum method. Ghorui *et al.* [8] studied the rotational bands and electromagnetic transitions of some even-even Neodymium nuclei in J-Projected Hartree-Fock Model Kassim *et al.* [9], [10] have

been studied nuclear structure of even-even $^{172-182}\text{Hf}$ isotopes by interacting Boson Model (IBM-1).

II. MATERIALS AND METHOD

Theoretical basics of Sood semi-empirical formula: The shape of atomic nuclei with a number of protons and neutrons exterior of closed shells is axially symmetric and permanently deformed [1]. The rotational energy band of an axially symmetric spinner is dependence on $J(J+1)$ if we take adiabatic approximations for the motions of all the particles in the nucleus. The low-lying energy states of even-mass rotational nuclei are $J^\pi = 0^+, 2^+, 4^+, 6^+$, etc. The (+) indicate for even parity state and rotational energy levels are given by the expression [4]:

$$E_J = AJ(J+1) \quad (1)$$

where symbol A is rotational parameter which is inversely proportional to the moment of inertia $\mathfrak{I}$, and $A = \hbar^2/2\mathfrak{I}$. The moment of inertia is expected to be constant where an approximation for the energy ratio $E(4^+)/E(2^+) = 10/3$, $E(6^+)/E(2^+) = 7$, $E(8^+)/E(2^+) = 12$. Deviations by a few percent from these ratios are thought to be a coupling of vibrational and rotational modes of the nuclei. This coupling depends on J^2 and constitutes changes in rotational energy from the first-order perturbation theory. However, for fitting the first three or four excited states, the first-order correction term can be added to equation (1) where the rotational energy [4], [11], [12]:

$$E_J = AJ(J+1) - BJ^2(J+1)^2 \quad (2)$$

Even with the addition of correction terms in equation (2), at the higher rotational state where $J > 10$ the approximation will deviate significantly [2] from experimental values. Stephens *et al.* [13] and Singla *et al.* [14] added second-order $CJ^3(1+J)^3$ and third-order $DJ^4(1+J)^4$ correction terms respectively to account for the deviations in energy bands for even mass actinide nuclei. The equation of rotational energy in terms of power series [15]:

$$E_J = AJ(1+J) + BJ^2(1+J)^2 + CJ^3(1+J)^3 + DJ^4(1+J)^4 \quad (3)$$

Assuming that the coefficient parameters are constant through each successive order of correction terms, where $CB^{-1} = DC^{-1} = N$ (BA^{-1}) and Sood develop a semi-empirical formula for rotational energy by summing of infinite series in equation (3):

$$E_J = AJ(1+J) \left[\frac{1 + (N-1)\left(\frac{B}{A}\right)J(1+J)}{1 + N\left(\frac{B}{A}\right)J(1+J)} \right] \quad (4)$$

where ratio $B/A = (10-3R) / (200-18R)$, where $R = E(4^+)/E(2^+)$. The constant N was obtained by Sood [15] via empirical method was:

$$N = 2.85 - 0.05J \quad (5)$$

In this study, Sood's semi-empirical formula (SSEF) has been utilized to calculate the G.S. bands of even mass hafnium nucleus of $A=168$ with spin state up to 18^+ . Hafnium isotope was chosen because it lies between deformed rare-earth nuclei and double magic ^{208}Pb nucleus [16].

III. RESULTS AND DISCUSSION

The calculated values of energy levels based on Sood's semi-empirical formula are presented with experimental values of low-lying rotational energy levels where $K^\pi = 0^+$ and J^π take even values from 0^+ up to 18^+ . We note that the percentage of deviation between calculated and experimental values is very small in ^{168}Hf isotope relative to ^{166}Hf [17]. The deviation of experimental and theoretical calculation of ^{168}Hf yrast level $2^+, 4^+, 6^+, 8^+, 10^+, 12^+, 14^+, 16^+, 18^+$ are 0.21, 2.03, 2.61, 2.99, 3.48, 4.25, 7.08, 13.26 and 16.22 in percentage respectively. The discrepancy increases in higher spin states especially in lighter Hafnium isotopes.

To ease the presentation of data, numerical results extracted from nuclear data sheets are presented as red dots in a graph of the spin state against excitation energy in the KeV scale. The black squares represent calculated energy values based on Sood's semi-empirical formula. The purpose of lines connecting the points is only to guide the eyes because the spin state takes integer values. The error bars in experimental values are not visible due to relatively small values compared to observed energy scales.

TABLE I: EMPIRICAL [18] AND THEORETICAL CALCULATION OF YRASTS BANDS FOR ^{168}Hf NUCLEUS

A = 168 [16]			
J^π	Experiment E(J^π) [KeV]	Calculated E(J^π) [KeV]	Deviation %
0^+	0	0	0
2^+	124.10(5)	124.355	0.21
4^+	385.92(6)	393.767	2.03
6^+	757.29(7)	777.042	2.61
8^+	1213.70(15)	1249.988	2.99
10^+	1736.06(20)	1796.506	3.48
12^+	2306.09(23)	2404.028	4.25
14^+	2857.5(3)	3059.887	7.08
16^+	3310.4(3)	3749.289	13.26
18^+	3832.5(3)	4454.122	16.22

Fig. 1. Ground State Rotational Bands of Even-Even ^{168}Hf Isotopes.

From Fig. 1, we notice that for heavier Hafnium isotopes, the phenomenological Sood's semi-empirical formula fits the observed energies to the extent of remarkable agreement up

to spin state 18^+ which relate to data in table 1. However, in lower mass isotopes, systematic deviations are observed between calculated and experimental energies. For ^{168}Hf , the deviation becomes significant, starting from 14^+ states at 7.08 percent, while in ^{166}Hf [17], the deviation becomes significant from 10^+ states at 11.05 percent. The deviations from $J(J+1)$ rule in rotational energy bands are interpreted as the firmness of energy levels, or in other words, an increase in nuclear moment of inertia as the nucleus undergoes changing rotational angular velocity.

IV. CONCLUSIONS

Sood's semi-empirical formula is successfully applied to calculate the rotational spectrum of even-even ^{168}Hf nucleus. The results are sound with measured values within the prospects of the simple phenomenological fitting. The fitting perform very well for higher mass isotopes such as ^{168}Hf relative to lower mass ^{166}Hf [17]. The percentage of deviation between calculated and experimental values is very small in ^{168}Hf nucleus. The error percentage for the deviation of experimental and theoretical calculation of ^{168}Hf nucleus for yrast level 2^+ , 4^+ , 6^+ , 8^+ , 10^+ , 12^+ , 14^+ , 16^+ , 18^+ are 0.21, 2.03, 2.61, 2.99, 3.48, 4.25, 7.08, 13.26 and 16.22, respectively. The discrepancy increases in higher spin states especially in lighter Hafnium isotopes. These isotopes indicate a better phenomenological fitting model is required for predicting rotational energy levels in lower mass hafnium isotopes.

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DECLARATION OF STATEMENT

All authors are equally contributed to this work.

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